The Effect of Register Taps in the SAT-based Cryptanalysis of Stream Ciphers

Alexander A. Griffith

Department of Mathematics and Computer Science
Wittenberg University
Springfield, OH 45504
s11.agriffith@wittenberg.edu

Abstract. Crypto1 and Hitag2 are two well-known stream ciphers that are quite similar in structure. Despite their like-minded designs, they differ in their resistance to algebraic attacks. Both have been broken using SAT-solvers, but the fastest published attack against Hitag2 took over 6 hours, while Crypto1 was broken in under a minute. In this paper we experimentally examine how register taps and non-linear functions (NLFs) affect the security of stream ciphers against algebraic cryptanalysis, using modified versions of Crypto1 and Hitag2 as test subjects. Using these experiments, we demonstrate that the choice of register taps is an important element in the design of stream ciphers. Additionally, using data from manually created and random tap configurations, we create a score that relates two characteristics of a tap configuration (distance and regularity) to cipher security.

Keywords: Register taps, stream cipher, Crypto1, Hitag2, algebraic cryptanalysis, SAT-solver

1 Introduction

The design of stream ciphers has become more scrutinized since the advent of algebraic cryptanalysis. Much attention has been paid to the design of boolean functions used in stream ciphers, trying to prevent correlation and annihilator attacks used against them. The criterion that came out of this work is the notion of algebraic immunity[6], which assesses the affective non-linearity of the function, given its annihilators. However, while high algebraic immunity is necessary for cipher security, it is not sufficient, as experimental attacks using Gröbner bases and SAT-solvers have broken stream ciphers with functions having high algebraic immunity. The stream ciphers Crypto1 and Hitag2 are examples of this, as both have high algebraic immunity and yet have been broken using SAT-solvers[4][5][13]. Though both have been broken, there is a significant difference in the time needed to break the the two ciphers. This paper explores how

the non-linear function (NLF) taps (the register bits that are fed into the NLF) affect the time needed to break these ciphers using SAT-solvers.[1]

Contribution

In this paper we offer experimental results and analysis on the effect of NLF taps in the cryptanalysis of the stream ciphers Crypto1 and Hitag2 using SAT-solvers. Timed attacks on Crypto1, Hitag2, and four hybrid cryptosystems that explore different combinations of the two ciphers' NLFs and NLF taps are presented. Additionally, data from tests run on tap configurations that were manually created and randomly generated is used to develop a score that relates characteristics of tap configurations to their security. The paper is organized as follows. Section 2 provides a background for stream ciphers, algebraic cryptanalysis, and SAT-solvers, while Section 3 describes Crypto1 and Hitag2. In Section 4 we describe our experimental work on Crypto1 and Hitag2. Section 5 focuses on our experimental work on manually configured and randomly generated tap configurations, and in Section 6 we develop a security score for tap configurations. The paper is concluded in Section 7.

2 Stream Ciphers and Algebraic Cryptanalysis

In cryptography, encryption is the algorithmic process of making data unintelligible to those without secret information (a secret key). A variety of algorithms, called ciphers, exist for doing this. Stream ciphers are a branch of symmetric key (meaning both the sender and receiver must share the same secret key) encryption algorithms that operate on single bits of data. For stream ciphers, the secret key is a private bit array that is used in the encryption process. Stream ciphers generate a pseudo-random stream of bits which get XORed bit-by-bit with the plaintext (unencrypted message) to produce ciphertext (the encrypted message). Because they operate on single bits of data, they operate quickly and do not require prior knowledge of the message length, making them perfect for use in small electronic devices like mobile phones and smart cards.

Stream ciphers generally consist of some combination of linear feedback shift registers (LFSRs) and non-linear filter functions (NLFs). NLFs are typically used to combine bits from the register in a non-linear fashion to produce the keystream. Each time a new keystream bit is produced, the LFSR shifts by one spot and the empty spot is filled by a linear combination (a simple XOR) of some subset of the register.

For example, the simple stream cipher in Figure 1 uses a degree-three NLF to combine four bits of the register (register bits 2, 3, 5, and 7) in order to produce the keystream, which then gets XORed with a bit of plaintext. Each time a bit of keystream is produced, the register shifts one bit to the right, and

[1] This research is the continuation of work done at a Research Experience for Undergraduates (REU) sponsored by the National Science Foundation and jointly held by Northern Kentucky University and the University of Cincinnati.

the left-most place in the register is filled with the XOR of bits 1, 5, and 7. Consider what happens if we try to encrypt the message "01000001" and fill the register with a secret key of [0,0,1,1,0,1,1] from left to right. Then the first bit of keystream is $f(0,1,0,1) = 0*1*0+1*0*1+0*1+1*0+0*1+0 = 0$, which gets XORed with the first bit of plaintext, 0, to produce the ciphertext bit of 0. The register then shifts one bit to the right and the empty spot is filled with 0 + 0 + 1 = 1. Then the second keystream bit is generated in a similar fashion using $f(0,0,1,1)$ and the register is updated again. This continues until the entire plaintext message is encrypted. If we continued to the end, our keystream would be [0,1,1,0,0,1,1,1], meaning our plaintext message of "01000001" would be encrypted as "00100110." Since XOR is its own inverse, decryption works in a similar fashion. The receiver simply generates the same stream and XORs it with the ciphertext to recover the plaintext. Real stream ciphers are larger and more complex than this example. The ciphers we dealt with, for example, had 48-bit LFSRs, multiple NLFs, and initialization protocols that set up the register before generating keystream.

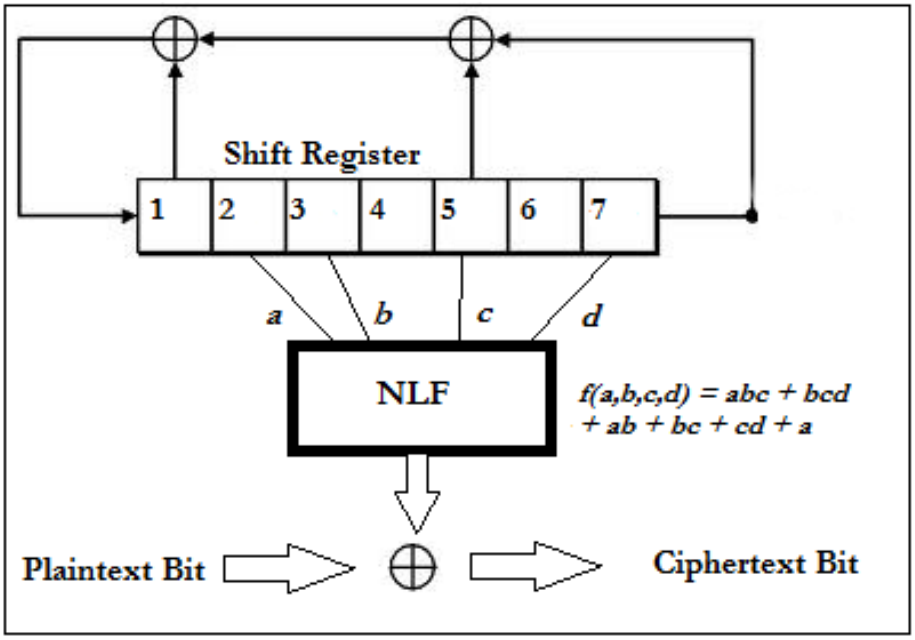


Fig. 1. A simple stream cipher

2.1 Algebraic Cryptanalysis

Cryptanalysis is the process of attempting to decrypt encrypted data without having access to the secret key. A variety of cryptanalytic techniques exist, but one that has gained much attention in recent years is algebraic cryptanalysis. Algebraic cryptanalysis aims at attacking ciphers by treating the cryptosystem as a constraint-satisfaction problem (CSP) and then attempting to solve that CSP. A CSP is a mathematics problem in which a set of objects must simultaneously satisfy given limitations. In the case of algebraic cryptanalysis the objects are bits of data and the constraints are the relationships between these bits as specified by the encryption algorithm. The cipher is modeled by a system of polynomial equations over some finite field, typically the finite field with two elements, $\mathbb{F}_2$, as there is a natural correspondence between bits of data and the set $\{0, 1\}$ with mod 2 arithmetic (addition corresponds to XOR, while multiplication corresponds to AND). As an example of this, the following is the polynomial system representing our earlier simple cipher's encryption of the plaintext "01000001" to the ciphertext "00100110." Solving this system would give us the secret key.

```
[   x[1]*x[3]*x[5] + x[3]*x[5]*x[6] + x[1]*x[3] + x[3]*x[5]
        + x[5]*x[6] + x[6] + x[17], // Keystream generation
    x[1] + x[3] + x[7] + x[8],  // Register update
    x[2]*x[4]*x[6] + x[4]*x[6]*x[7] + x[2]*x[4] + x[4]*x[6]
        + x[6]*x[7] + x[7] + x[18],
    x[2] + x[4] + x[8] + x[9],
    x[3]*x[5]*x[7] + x[5]*x[7]*x[8] + x[3]*x[5] + x[5]*x[7]
        + x[7]*x[8] + x[8] + x[19],
    x[3] + x[5] + x[9] + x[10],
    x[4]*x[6]*x[8] + x[6]*x[8]*x[9] + x[4]*x[6] + x[6]*x[8]
        + x[8]*x[9] + x[9] + x[20],
    x[4] + x[6] + x[10] + x[11],
    x[5]*x[7]*x[9] + x[7]*x[9]*x[10] + x[5]*x[7] + x[7]*x[9]
        + x[9]*x[10] + x[10] + x[21],
    x[5] + x[7] + x[11] + x[12],
    x[6]*x[8]*x[10] + x[8]*x[10]*x[11] + x[6]*x[8]
        + x[8]*x[10] + x[10]*x[11] + x[11] + x[22],
    x[6] + x[8] + x[12] + x[13],
    x[7]*x[9]*x[11] + x[9]*x[11]*x[12] + x[7]*x[9]
        + x[9]*x[11] + x[11]*x[12] + x[12] + x[23],
    x[7] + x[9] + x[13] + x[14],
    x[8]*x[10]*x[12] + x[10]*x[12]*x[13] + x[8]*x[10]
        + x[10]*x[12] + x[12]*x[13] + x[13] + x[24],
    x[8] + x[10] + x[14] + x[15],
    x[17],        // Because we assume knowledge of
    x[18] + 1,    // plaintext/ciphertext pairs we know
    x[19] + 1,    // the values of the keystream bits
    x[20],        // (keytstream = plaintext + ciphertext)
```

```
x[21],
x[22] + 1,
x[23] + 1,
x[24] + 1 ]
```

Attempts are then made to solve the polynomial system. A variety of methods exist for solving systems of polynomial equations. These include Gröbner basis algorithms (such as Faugère's F_4 [9] and F_5 [10] algorithms) which focus on expanding the ideal generated by the polynomials, eXtended Linearization algorithms (such as XL [3], MutantXL [7], and MXL2 [12]) which use linear algebra, and logic based SAT-solvers (such as MiniSAT [8] and CryptoMiniSAT [13]) since the work is typically done over $\mathbb{F}_2$. In general, finding the solutions of a polynomial system over a finite field is NP-complete. However, the polynomial systems that result from modeling ciphers are often very sparse (i.e., there are a large number of coefficients equal to zero) and highly structured, and therefore often permit successful cryptanalysis. While algebraic attacks have had limited success on other types of ciphers, stream ciphers have permitted a number of successful algebraic cryptanalysis attempts. Many of these successful attacks have used SAT-solvers as their method for solving. For a thorough introduction to algebraic cryptanalysis, see [1].

2.2 SAT-solvers in Cryptanalysis

SAT-solver programs try to determine whether or not a given set of boolean constraints has a solution. Conflict-driven SAT-solvers, such as MiniSat [8] (the program used for our experiments), use tree-based search algorithms with learning. MiniSat guesses the value of a variable and propagates the value throughout. If this guess causes a conflict with an earlier guess, MiniSat creates a new learned constraint, backtracks up the tree to the highest guess allowed by the new constraint, and changes the value of that guess. These learned constraints restrict the search space by trimming branches of the search tree, making it faster than brute force provided a solution exists. For an introduction to SAT-solvers, see Chapter 14 of [1].

SAT-solvers operate on constraints in their conjunctive normal form (CNF). In CNF, each variable and monomial from a polynomial system becomes a boolean variable. The disjunction (ORing) of boolean variables is called a clause, and CNF is the conjunction (ANDing) of these clauses. To use a SAT-solver on a polynomial system, the polynomials must first be converted to CNF. A variety of programs exist for this. Soos [14] even has a program, Grain of Salt, that translates stream ciphers directly to CNF. We used a Perl script written by Jeremy Erickson[2] to convert our polynomials into CNF. The CNF of the polynomial system from our earlier example can be found in Appendix A.

[2] A former NKU/UC REU participant. Now a Computer Science graduate student at the University of North Carolina.

3 Crypto1 and Hitag2

Crypto1 and Hitag2 are two stream ciphers that have received a lot of negative attention in recent years, as both ciphers have been successfully attacked on multiple occasions, often with algebraic cryptanalysis. Both ciphers have been used in industrial applications making their lack of security more glaring.

3.1 Crypto1

Crypto1 is a proprietary stream cipher used in the MiFare Classic smart card and is manufactured by NXP (formerly Phillips) Semiconductors. The MiFare Classic smart card is used in a variety of public transportation payment systems, most notably the London Oyster card. Since MiFare Classic has been reverse engineered[11], the Crypto1 cipher has been attacked and broken numerous times. Some of these attacks used SAT-solver based algebraic cryptanalysis[4][13]. These attacks broke Crypto1 in 200 seconds [4] and 40 seconds [13] respectively. The Crypto1 cipher consists of a 48-bit LFSR and six NLFs that are combined to generate keystream. The LFSR has an update function $f : \mathbb{F}_2^{18} \to \mathbb{F}_2$ that simply XORs 18 bits of register to produce the new bit. There are 20 bits of register that are fed into five degree-3 NLFs $g_i : \mathbb{F}_2^4 \to \mathbb{F}_2, i = 0, ..., 4$. The outputs of these functions are subsequently fed into one degree-4 NLF $g_5 : \mathbb{F}_2^5 \to \mathbb{F}_2$ that produces the keystream output (see Figure 1 of Appendix B).

The cipher is initialized using a 48-bit secret key, a 32-bit serial number, and two 32-bit initialization vectors (IVs) in the following manner. The secret key is used to fill the register. Then, for 32 clocks, the LFSR shifts one position to the left and one bit of serial is XORed with one bit from the Tag IV and the output from f to fill the new position in the register. The cipher continues with a second initialization phase for 32 more clocks. The LFSR shifts one position to the left, and one bit of Reader IV gets XORed with one bit of keystream (the output of $g_5(g_0, ..., g_4)$) and the output from f, to fill the empty space in the register. After this, initialization is complete. Then keystream generation begins. Keystream generation operates for n-clocks, where n is the number of plaintext bits to encrypt. The register is updated exclusively by the linear feedback function f for the rest of the encryption, and plaintext bits are encrypted by XORing them with a keystream bit produced at each clock. The keystream is generated using the functions $g_0, ..., g_5$ as described earlier. The complete initialization and encryption process of Crypto1 is described in Algorithm 1.

3.2 Hitag2

Hitag2 is a stream cipher very similar to Crypto1. Hitag2 is used in RFID car lock remotes and is also manufactured by NXP Semiconductors. These remotes are used in variety of cars, including some models made by Ford, GM, and Volvo. Though attacks against Hitag2 are not as common as those against Crypto1, it has been broken since its design became public, including attacks using SAT-solvers [5] [13].

Algorithm 1 Crypto1

{$x_1 ... x_{48}$ are register bits}
{n is the number of plaintext bits to encrypt}
{$g_0 = g_3 = acd + bcd + ab + ac + ad + bc + cd + a + b$}
{$g_1 = g_2 = g_4 = abc + acd + bcd + ab + ac + ad + bc + bd + c$}
{$g_5 = abce + bcde + abd + abe + acd + ade + bde + ac + ad + ae + de + a + e$}

{First initialization phase}

for i in 1..32 **do**
t := $x_1 \oplus x_6 \oplus x_{10} \oplus x_{11} \oplus x_{13} \oplus x_{15} \oplus x_{16} \oplus x_{18} \oplus x_{20} \oplus x_{25} \oplus x_{26} \oplus x_{28} \oplus x_{30} \oplus x_{36} \oplus x_{40} \oplus x_{42} \oplus x_{43} \oplus x_{44} \oplus$ Serial[i] $\oplus$ TagIV[i]
$(x_1, ..., x_{48}) := (x_2, ..., x_{48}, t)$
end for

{Second initialization phase}

for i in 1..32 **do**
t := $x_1 \oplus x_6 \oplus x_{10} \oplus x_{11} \oplus x_{13} \oplus x_{15} \oplus x_{16} \oplus x_{18} \oplus x_{20} \oplus x_{25} \oplus x_{26} \oplus x_{28} \oplus x_{30} \oplus x_{36} \oplus x_{40} \oplus x_{42} \oplus x_{43} \oplus x_{44} \oplus$ ReaderIV[i] $\oplus g_5(g_0(x_{10}, x_{12}, x_{14}, x_{16}), ..., g_4(x_{42}, x_{44}, x_{46}, x_{48}))$
$(x_1, ..., x_{48}) := (x_2, ..., x_{48}, t)$
end for

{Keystream generation}

for i in 1..n **do**
t := $x_1 \oplus x_6 \oplus x_{10} \oplus x_{11} \oplus x_{13} \oplus x_{15} \oplus x_{16} \oplus x_{18} \oplus x_{20} \oplus x_{25} \oplus x_{26} \oplus x_{28} \oplus x_{30} \oplus x_{36} \oplus x_{40} \oplus x_{42} \oplus x_{43} \oplus x_{44}$
$k_i := g_5(g_0(x_{10}, x_{12}, x_{14}, x_{16}), ..., g_4(x_{42}, x_{44}, x_{46}, x_{48}))$
$(x_1, ..., x_{48}) := (x_2, ..., x_{48}, t)$
end for

Hitag2 also consists of a 48-bit LFSR and six NLFs that are combined to generate keystream. The LFSR's feedback function $f : \mathbb{F}_2^{16} \to \mathbb{F}_2$ XORs 16 register bits to produce the new register bit. There are 20 bits of register that are fed into five degree-3 NLFs $g_i : \mathbb{F}_2^4 \to \mathbb{F}_2, i = 0, ..., 4$. The outputs of these functions are subsequently fed into one degree-4 NLF $g_5 : \mathbb{F}_2^5 \to \mathbb{F}_2$ that produces the keystream output.

Hitag2's initialization process differs slightly from Crypto1. Hitag2 is initialized using a 48-bit secret key, a 32-bit serial number, and a 32-bit IV. Its LFSR is initially filled with the 32-bit serial and the 16 lowest ranking bits of key. For 32 clocks, the LFSR shifts one position to the left, and the empty register bit is filled with the XOR or three bits: one bit of IV, one bit of key not already in the register, and one bit of output from the NLFs. Then keystream generation begins. The LFSR is updated by f, and keystream is generated using $g_0, ..., g_5$. The encryption algorithm is listed in Algorithm 2, and a diagram of the cipher can be found in Figure 2 of Appendix B.

Algorithm 2 Hitag2

{$x_1 ... x_{48}$ are register bits}
{n is the number of plaintext bits to encrypt}
{$g_0 = g_4 = abc + ac + bc + ad + a + b + d + 1$}
{$g_1 = g_2 = g_3 = abd + acd + bcd + ab + ac + bc + a + b + d + 1$}
{$g_5 = abce + abde + acd + ade + bcd + bce + cde + ab + bc + bd + be + ce + de + b + d + 1$}

{Initialization phase}

for i in 1..32 **do**
t := IV[i] $\oplus$ KEY[i+16] $\oplus g_5(g_0(x_2, x_3, x_5, x_6), g_1(x_8, x_{12}, x_{14}, x_{15}), g_2(x_{17}, x_{21}, x_{23}, x_{26}), g_3(x_{28}, x_{29}, x_{31}, x_{33}), g_4(x_{34}, x_{43}, x_{44}, x_{46}))$
$(x_1, ..., x_{48}) := (x_2, ..., x_{48}, t)$
end for

{Keystream generation}

for i in 1..n **do**
t := $x_1 \oplus x_3 \oplus x_4 \oplus x_7 \oplus x_8 \oplus x_9 \oplus x_{17} \oplus x_{23} \oplus x_{24} \oplus x_{27} \oplus x_{31} \oplus x_{42} \oplus x_{43} \oplus x_{44} \oplus x_{47} \oplus x_{48}$
$k_i := g_5(g_0(x_2, x_3, x_5, x_6), g_1(x_8, x_{12}, x_{14}, x_{15}), g_2(x_{17}, x_{21}, x_{23}, x_{26}), g_3(x_{28}, x_{29}, x_{31}, x_{33}), g_4(x_{34}, x_{43}, x_{44}, x_{46}))$
$(x_1, ..., x_{48}) := (x_2, ..., x_{48}, t)$
end for

4 Experimental Comparison of Crypto1 and Hitag2

The main goal for this portion of our research was to determine if the difference in register taps was the main factor in the security desparity between the two ciphers. Since the non-linear functions (NLFs) and register taps (the inputs of the NLFs) were the most apparent differences between the ciphers, these were the variables we concentrated on. To analyze the effect of NLFs and register taps on the stream ciphers Crypto1 and Hitag2, SAT-solver timing tests were conducted on the following systems:

- Crypto1
- Hitag2
- Crypto1 with Hitag2 NLFs
- Hitag2 with Crypto1 NLFs
- Crypto1 with Hitag2 Taps
- Hitag2 with Crypto1 Taps

For each system, ten instances were created and tested. Each of these instances used a different randomly generated key, IV, and serial to encrypt 56 bits of plaintext. For Hitag2 this is not a practical attack (as noted in [5] encrypting a message of this length would require additional initialization vectors), but 56 bits were chosen to ensure a unique solution for each system, and to stay consistent with the attacks in [13]. The encryptions and polynomial systems were

generated using MAGMA[3] [2], converted to CNF, and then solved using MiniSat 2.0[4]. All tests were run on Wittenberg University's WARP2 next generation computing cluster.

The SAT-solver running times (in seconds) of Crypto1, Hitag2, and hybrids are listed in Table 1. The $*$ symbol indicates that the given test was killed after 14 days. Each system used the same ten seeds to generate the random keys, IVs, and serials.

Table 1. SAT-solving times for Crypto1, Hitag2, and Hybrid Systems with swapped NLFs(in seconds)

Cipher	Crypto1	Hitag2	C1 w/ H2 NLFs	H2 w/ C1 NLFs	C1 w/ H2 Taps	H2 w/ C1 Taps
Test 1	26.88	$*$	516.12	157,367.20	2958.50	5994.41
Test 2	127.12	726,967.74	206.65	210,117.45	63,960.98	1052.12
Test 3	276.56	799,127.10	724.55	108,557.26	5860.80	3677.58
Test 4	153.89	$*$	145.93	53,260.86	$*$	4131.03
Test 5	1752.91	$*$	448.61	46,375.79	$*$	285.55
Test 6	152.72	$*$	50.64	$*$	850,311.27	6092.15
Test 7	712.28	$*$	1059.98	32,949.12	30,202.11	862.00
Test 8	380.74	102,639.52	217.46	$*$	57,846.76	448.47
Test 9	1340.23	$*$	362.25	98,052.59	50,286.20	4372.88
Test 10	86.63	$*$	1361.33	101,613.74	314,860.44	1262.57
Mean	500.90	N/A	509.35	N/A	N/A	2817.91

Based on this data, it appears that the greatest difference between Crypto1 and Hitag2 in terms of security is the difference in register taps, as expected. Swapping NLFs had relatively small effects (increasing the average runtime by nine seconds in Crypto1 and causing five more tests to finish in less than 14 days in Hitag2), but swapping taps had a large impact. Swapping the taps caused Crypto1 to go from having an average runtime of about 8 minutes, to having 2 tests fail to finish in under fourteen days. Similarly, Hitag2 only had three tests finish in under fourteen days, but using Crypto1 taps, all 10 finished with an average runtime of about 47 minutes.

5 Manual and Random Tap Configurations

Based on our results with Crypto1, Hitag2, and the four hybrid systems, we discovered the choice of register taps to be an important factor in the security of

[3] Special thanks to the Computational Algebra Group at the University of Sydney for providing MAGMA for this project.

[4] Available at http://minisat.se/MiniSat.html

stream ciphers. Since the choice of taps is so important, we wanted to quantify how different aspects of tap configurations affected security. In order to do this, we ran tests on cryptosystems following Crypto1's protocol but with manually chosen taps.

5.1 Manually Configured Taps

The characteristics we chose to focus on were regularity (whether the taps followed a regular pattern), distance (how far apart the taps were), and register position (whether the taps were concentrated in the left, middle, or right portion of the register). Additional details on these characteristics can be found in Section 6.1. Tests were conducted in the same manner as those on Crypto1 and Hitag2, and were run on the following systems:

- Left-adjusted Crypto1—every other tap is used with the collection beginning on the left side of the register
- Center-adjusted Crypto1—every other tap is used with the collection of used taps centered in the register
- Left Consecutive—the twenty left-most taps in the register are used
- Center Consecutive—the twenty taps in the center of the register are used
- Right Consecutive—the twenty right-most taps in the register are used
- 2-2 Taps—bits in the register alternate between two consecutive taps and two consecutive non-taps
- 1-2-1 Taps—the number of register bits between taps alternates between one and two
- 19-1 Taps—the 19 left-most taps and the right-most tap
- 10-10 Taps—the ten left-most taps, a space, and then the next ten taps

A visualization of these tap configurations can be found in Appendix C. The results are summarized in the Table 2.

Table 2. SAT-solving times for Crypto1 with manually configured taps (in seconds)

Taps	Crypto1	Left C1	Mid C1	Left Consec	Mid Consec	Right Consec	2-2 Taps	1-2-1 Taps	19-1 Taps	10-10 Taps
Test 1	26.88	132.12	226.23	9.02	9.98	0.96	3413.63	52,049.16	9.04	2.05
Test 2	127.12	2808.10	451.39	24.20	2.29	0.24	9799.41	29,108.20	83.46	1.09
Test 3	275.56	80.08	136.66	1.46	0.84	1.68	894.08	5311.77	3.59	1.79
Test 4	153.89	616.59	13.19	1.92	3.13	1.61	24,364.80	146,676.02	34.85	4.50
Test 5	1752.91	1273.40	135.61	2.22	6.10	3.99	26,621.50	16,481.04	24.6	2.58
Test 6	152.72	274.68	56.80	9.95	2.60	1.29	563.35	28,874.82	18.92	3.54
Test 7	712.28	434.07	62.47	3.51	43.18	18.76	7377.13	7107.00	175.18	3.61
Test 8	380.74	117.08	589.71	5.00	1.18	1.77	1962.62	42,150.36	5.81	2.15
Test 9	1340.23	492.01	329.37	3.84	3.99	1.02	604.16	103,569.28	33.87	9.31
Test 10	86.63	2123.95	846.03	2.22	5.89	5.02	16,178.53	19,349.93	27.41	1.73
Mean	500.90	835.21	284.75	6.33	7.92	3.652	9177.92	45,067.76	41.67	3.24

This data indicated that distance was positively correlated with security and that register position had little effect. It also suggested that regularity was

negatively correlated with security. To further explore this, we tested randomly generated tap configurations.

5.2 Randomly Generated Taps

We randomly generated taps for six different tap configurations (A-F) and ran tests on them in the same fashion as our tests above. These tests used Crypto1's protocol and functions and the results are shown in the Table 3.

Table 3. SAT-solving times for Crypto1 with randomly generated taps (in seconds)

Taps	Rand A	Rand B	Rand C	Rand D	Rand E	Rand F
Test 1	622,821.10	39,462.55	49,157.27	663,549.73	423,270.91	29,106.78
Test 2	14,242.05	23,763.62	72,341.33	*	589,014.20	210,869.40
Test 3	391,570.56	1964.12	3714.08	180,678.58	412,970.57	446,532.26
Test 4	50,378.07	13,024.08	763,204.95	658,268.54	192,860.62	62,713.09
Test 5	103,576.18	9887.10	12,915.87	*	620,232.68	611,247.11
Test 6	736,915.77	662,668.36	738,421.84	1,028,884.23	258,563.13	197,543.79
Test 7	162,965.86	5474.42	335,768.48	28,760.83	32,282.07	69,156.34
Test 8	84,086.51	57,873.49	172,888.32	40,447.96	*	11,764.21
Test 9	138,650.05	97,395.13	155,567.60	*	283,487.27	264,012.74
Test 10	575,653.63	15,869.65	2569.52	*	*	32,328.55
Mean	288,085.98	92,738.25	230,654.93	*	*	193,527.43

These results further strengthened our notion that distance was positively correlated with security and regularity was negatively correlated with security, as the random tap configurations had more distance and less regularity, and took significantly longer to break. To better quantify this effect, we decided to develop a security score that assigns a number to tap configurations in order to predict security.

6 A Security Score for Tap Configurations

In order to quantify the ideas we saw in our experimental results, we developed a score that predicts security based on tap configuration characteristics. From our experimental results it was clear that distance and regularity subscores should be part of this score. However, capturing these qualities (especially regularity) in one measure proved difficult[5], so we ran a linear regression on six subscores that each help describe the distance and regularity characteristics of a tap configuration. These subscores were computed using a Java program we wrote which can be found in Appendix D. Descriptions of the subscores follow.

[5] Our first attempt at a security score was tested on a configuration that was created to have a Monte Carlo maximum score. All ten tests finished in an average of 2 days.

6.1 Distance

For the notion of distance, we wanted a measurement of how much spread there was between register taps. To do this, we decided to measure the distance of individual taps from a baseline configuration, and sum these distances. As baselines, we chose the Right Consecutive and Left Consecutive configurations. The reason for these baselines is the close spacing of the taps (they are consecutive), and the lack of security these tap configurations offered. We chose two baseline configurations to account for differences in register position, using the minimum of the two scores calculated.

As an example of how we calculated distance, consider the Crypto1 tap configuration: 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48. Since the Left Consecutive taps are 1 through 20, and the Right Consecutive taps are 29 through 48, our distance scores are:

$$LeftDistance = |1-10| + |2-12| + |3-14| + ... + |19-46| + |20-48| = 370$$

$$RightDistance = |29-10| + |30-12| + |31-14| + ... + |47-46| + |48-48| = 190$$

So Crypto1's final distance score would be the minimum of these two scores, which is 190.

6.2 Repetition

Regularity is a concept that is easy to understand, but difficult to quantify. We wanted a measure that shows whether a configuration has a regular, evenly-spaced tap pattern or not. Our first method of doing this, repetition, was to look at the four taps used in each of the five NLFs and see how many times those taps were used together in an NLF. For each set of four taps, we counted how many times all four those taps appeared in the same function, how many times three appeared in the same function, and how many times two appeared in the same function after 48 clocks (one complete cycle through the register). Instances where all four appeared in the same function were weighted with a four, three with a three, and two with a two. We did this for all five sets of taps. The result was a score that reflected our intentions as patterned configurations like Crypto1 and our Consecutive taps, had the highest regularity scores (183), and unpatterned configurations, like Hitag2, had a much lower score (109). The complete algorithm can be found in the getRepetitionScore() method of our Java source code in Appendix D.

6.3 Space Count

Another measure of regularity was our space count. This measure simply counted the number of different spaces between register taps in each tap configuration. For example, Crypto1 had one space between every tap, so the number of distinct spacings (our space count) was one. Our 1-2-1 taps however, alternated between

having one and two spaces between taps, so the number of distinct spacings was two. The complete algorithm can be found in the spaceCount() method of our Java source code in Appendix D.

6.4 Spacing Deviation

Spacing deviation was another measure we used to capture the regularity of a tap configuration. In order to calculate our spacing deviation, we took the set of spacings and computed the standard deviation of that set. So evenly spaced configurations like Crypto1 had a spacing deviation of 0, while sporadically spaced configurations like Hitag2 (1.808) had higher spacing deviations. The complete algorithm can be found in the spaceDev() method of our Java source code in Appendix D.

6.5 Span Deviation

Span deviation was our last score that focused solely on regularity. We defined a span as the length of a set of continuous taps. Our span deviation was then the standard deviation of the set of spans for each tap configuration. For example our consecutive taps had one span of 20 taps and Crypto1 had 20 spans of 1 tap, but both of their span deviations were 0. The complete algorithm can be found in the spanDev() method of our Java source code in Appendix D.

6.6 Inclusion

Our final measure was inclusion, which incorporates both distance and regularity. Our inclusion score simply shifts the register four times, and each time counts the number of original taps used in the NLFs. This characteristic is negatively correlated with security, as configurations that were solved quickly had higher inclusion scores. The complete algorithm can be found in the getDiffusionScore() method of our Java source code in Appendix D.

6.7 Fitting the Security Score

After computing the individual subscores, our next step was combining them in a meaninful way. Using the statistics software MiniTab, we ran a linear regression with our subscores as the independent variables, and the natural log-adjusted (mean) time[6] as our dependent variable. The result was a score (indicated by the Final Score column in the table) that combined our subscores in the the following manner:

$$FinalScore = 8.05 + .014 * Distance - .0081 * Repetition + .6 * SpaceCount$$

$$+1.1 * SpaceDev - .573 * SpanDev - .0939 * Inclusion$$

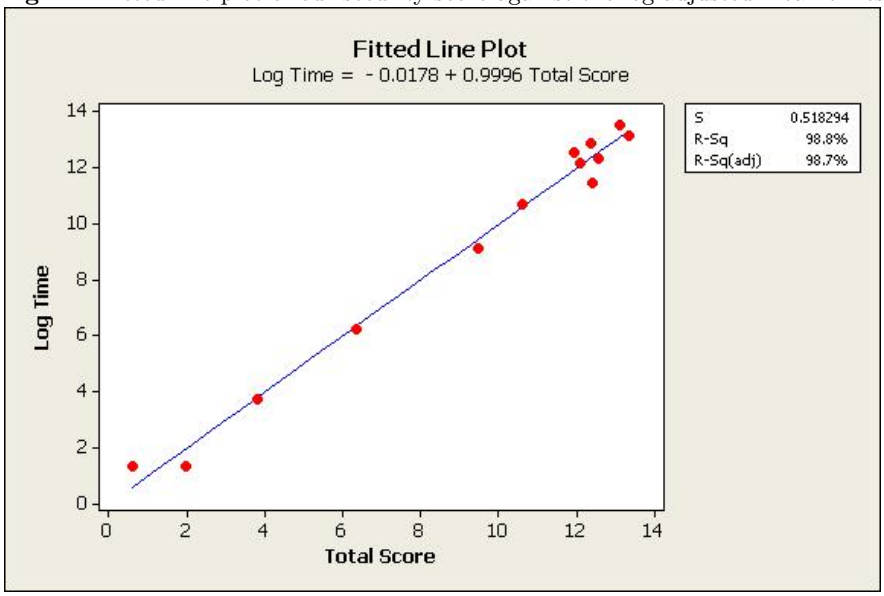
Table 4. Configuration subscores, final score, and log-adjusted time

	Distance Regularity	Inclusion Space	Count Space	Dev Span	Dev	Final Score	Log Time
Right Con	0	183	70	1	0	0	0.5947	1.2953
Crypto1	190	183	37	1	0	0	6.3534	6.2164
2-2 Taps	280	162	37	2	0.9986	0	9.4820	9.1246
1-2-1 Taps	280	162	19	2	0.4993	0	10.6229	10.7159
Hitag2	230	109	29	5	1.8080	0.4949	12.3692	12.8600
Rand A	277	125	36	6	1.8018	2.0494	11.9428	12.5710
Rand B	250	102	31	6	1.6560	1.4025	12.4309	11.4375
Rand C	255	108	29	6	1.4643	1.1513	12.5731	12.3487
Rand D	263	117	28	6	1.8727	1.1923	13.1319	13.5197
Rand E	266	104	33	6	2.1353	0.7454	13.3546	13.1677
Rand F	271	64	27	4	1.2230	0.7458	12.1083	12.1732
19-1 Taps	28	168	66	2	6.2523	9.000	3.8043	3.7298
10-10 Taps	10	182	66	2	0.2233	0	1.9640	1.2953

The data for this regression can be found in the Table 4.

As seen in the Figure 2, our regression analysis resulted in a very good fit. Using the equation of $LogTime = -0.0178 + 0.9996 * FinalScore$, we obtained a fit with an R-squared value of 98.8%. These six qualities have an apparent relationship with cipher security.

Fig. 2. A fitted line plot of our security score against the log-adjusted mean times



[6] For configurations where a mean was not possible (as not all instances finished), we used our cutoff time (2 weeks) as the time for those tests.

6.8 Our Security Score as a Predictor

One of our goals was to develop our security score as a tool for stream cipher design. To test our score in this regard, we included in our Java program a method to randomly generate tap configurations. We generated 50 million configurations, computed their scores, and kept track of the maximum score. The highest score we obtained was 15.25 for the configuration 1, 2, 8, 9, 10, 13, 14, 18, 19, 20, 25, 26, 27, 38, 39, 41, 42, 44, 46, 47, and we ran tests on this configuration, hoping to demonstrate that this configuration was secure.

If the data behaves as the score predicts, then these tests should run in average of $e^{15.25}$ seconds, or about 49 days (though results this extreme are unlikely as our cutoff times skew the end behavior of our security score). Our final results are still pending, but early data shows that it is already our most secure configuration, indicating that our security score is a useful tool in stream cipher analysis and design.

7 Conclusion and Future Work

Register taps play a major role in stream cipher security with regards to SAT-based Cryptanalysis. In our experiments with Crypto1, Hitag2, and the four hybrid systems we determined that the difference in taps was the biggest factor in the two ciphers' security difference. Furthermore, by examining our manually configured and randomly generated taps we identified quantifiable characteristics (distance and regularity) that are highly correlated with security, and developed a security score based on these characteristics. In the future, it is our hope that this kind of analysis will be carried out in stream cipher design. Experimental cryptanalysis reduces the chance that weak ciphers like Crypto1 will be used in practical applications.

References

1. G. Bard. *Algebraic Cryptanalysis*. Springer, New York, NY, 2009.

2. W. Bosman, J. Cannon, and C. Playhoust. *The Magma algebra system. I. The User Language.* In: J. of Symbolic Comput., 24(3-4), pp. 235-265, 1997.

3. N. Courtois, A. Klimov, J. Patarin, and A. Shamir. *Efficient Algorithms for Solving Overdefined Systems of Multivariate Polynomial Equations*. In: EUROCRYPT 2000, LNCS 1807, pp. 392-497.

4. N.T. Courtois, K. Nohl, and S. O'Neil. *Algebraic Attacks on the Crypto-1 Stream Cipher in MiFare Classic and Oyster Cards*. Cryptology ePrint Archive, Report 2008/166, 2008. http://eprint.iacr.org/.

5. N.T. Courtois, S. O'Neil, and J.J. Quisquater. *Practical Algebraic Attacks on the Hitag2 Stream Cipher*. In: P. Samarati et al. (Eds.): ISC 2009, LNCS 5735, pp. 167-176, 2009.

6. D.K. Dalai, K.C. Gupta, and S. Maitra. *Notion of Algebraic Immunity and Its evaluation Related to Fast Algebraic Attacks*. Cryptology ePrint Archive, Report 2006/018, 2006. http://eprint.iacr.org/.

7. J. Ding, J. Buchmann, M.S.E. Mohamed, W.S.A Mohamed, and R.P. Weinmann. *MutantXL*. In: Proceedings of the 1st international conference on Symbolic Computation and Cryptography (SCC 2008), LMIB, pp. 23-32.

8. N. Eén N. Sörensonn. *An extinsible SAT-solver*. In: E. Giunchiglia and E. Tacchella (Eds.): SAT. LNCS 2919, pp. 502-518, 2003.

9. J.C. Faugère. *A New Efficient Algorithm for Computing Gröbner Bases (F_4*. In: Journal of Pure and Applied Algebra, 139, pp. 61-88, 1999.

10. J.C. Faugère. *A New Efficient Algorithm for Computing Gröbner Bases Without Reduction to Zero (F_5)*. In: Proceedings of ISSAC, pp. 75-83, 2002.

11. F.D. Garcia, G. de Koning Gans, R. Muijrers, P. van Rossum, R. Verdult, R.W. Schreur, and B. Jacobs. *Dismantling MIFARE Classic*. In: S. Jajodia, and J. Lopez (Eds.): ESORICS 2008, LNCS 5283, pp. 97-114, 2008.

12. M.S.E. Mohamed, W.S.A.E. Mohamed, J. Ding, and J. Buchmann. *MXL2: Solving Polynomial Equations over GF(2) Using an Improved Mutant Strategy*. In: J. Buchmann and J. Ding (Eds.): PQCrypto 2008, LNCS 5299, pp. 203-215, 2008.

13. M. Soos, K. Nohl, and C. Castelluccia. *Extending SAT Solvers to Cryptographic Problems*. In: Oliver Kullmann (Ed.): LNCS 5584, pp. 244-257, 2009.

14. M. Soos. *Grain of Salt — An Automated Way to Test Stream Ciphers through SAT Solvers*. (2010). http://planete.inrialpes.fr/ soos/GrainOfSalt/index.php

A CNF of our Simple Cipher Example

There are 73 boolean variables and 502 clauses. Each number is a boolean variable (with '-' indicating the negation of that variable) and each line is a clause. Each variable in a clause is ORed together and each clause is ANDed together. The variables 1 through 24 correspond to x[1] through x[24].

```
p cnf 73 502
25 0
1 -26 0  // 26 corresponds the monomial x[1]*x[3]*x[5]
3 -26 0  // 3 is True (x[3] = 1) or 26 is False (the monomial = 0)
5 -26 0
26 -1 -3 -5 0
3 -27 0
5 -27 0
6 -27 0
27 -3 -5 -6 0
1 -28 0
3 -28 0
28 -1 -3 0
3 -29 0
5 -29 0
29 -3 -5 0
5 -30 0
6 -30 0
30 -5 -6 0
-26 27 28 29 30 31 0
-26 -27 -28 29 30 31 0
-26 -27 -28 -29 -30 31 0
-26 -27 -28 -29 30 -31 0
-26 -27 -28 29 -30 -31 0
-26 -27 28 -29 30 31 0
-26 -27 28 -29 -30 -31 0
-26 -27 28 29 -30 31 0
-26 -27 28 29 30 -31 0
-26 27 -28 -29 30 31 0
-26 27 -28 -29 -30 -31 0
-26 27 -28 29 -30 31 0
-26 27 -28 29 30 -31 0
-26 27 28 -29 -30 31 0
-26 27 28 -29 30 -31 0
-26 27 28 29 -30 -31 0
26 -27 28 29 30 31 0
26 -27 -28 -29 30 31 0
26 -27 -28 -29 -30 -31 0
26 -27 -28 29 -30 31 0
```

```
26 -27 -28 29 30 -31 0
26 -27 28 -29 -30 31 0
26 -27 28 -29 30 -31 0
26 -27 28 29 -30 -31 0
26 27 -28 29 30 31 0
26 27 -28 -29 -30 31 0
26 27 -28 -29 30 -31 0
26 27 -28 29 -30 -31 0
26 27 28 -29 30 31 0
26 27 28 -29 -30 -31 0
26 27 28 29 -30 31 0
26 27 28 29 30 -31 0
-31 6 17 0
-31 -6 -17 0
31 -6 17 0
31 6 -17 0
-1 3 7 8 0
-1 -3 -7 8 0
-1 -3 7 -8 0
-1 3 -7 -8 0
1 -3 7 8 0
1 -3 -7 -8 0
1 3 -7 8 0
1 3 7 -8 0
2 -32 0
4 -32 0
6 -32 0
32 -2 -4 -6 0
4 -33 0
6 -33 0
7 -33 0
33 -4 -6 -7 0
2 -34 0
4 -34 0
34 -2 -4 0
4 -35 0
6 -35 0
35 -4 -6 0
6 -36 0
7 -36 0
36 -6 -7 0
-32 33 34 35 36 37 0
-32 -33 -34 35 36 37 0
-32 -33 -34 -35 -36 37 0
-32 -33 -34 -35 36 -37 0
```

```
-32 -33 -34 35 -36 -37 0
-32 -33 34 -35 36 37 0
-32 -33 34 -35 -36 -37 0
-32 -33 34 35 -36 37 0
-32 -33 34 35 36 -37 0
-32 33 -34 -35 36 37 0
-32 33 -34 -35 -36 -37 0
-32 33 -34 35 -36 37 0
-32 33 -34 35 36 -37 0
-32 33 34 -35 -36 37 0
-32 33 34 -35 36 -37 0
-32 33 34 35 -36 -37 0
32 -33 34 35 36 37 0
32 -33 -34 -35 36 37 0
32 -33 -34 -35 -36 -37 0
32 -33 -34 35 -36 37 0
32 -33 -34 35 36 -37 0
32 -33 34 -35 -36 37 0
32 -33 34 -35 36 -37 0
32 -33 34 35 -36 -37 0
32 33 -34 35 36 37 0
32 33 -34 -35 -36 37 0
32 33 -34 -35 36 -37 0
32 33 -34 35 -36 -37 0
32 33 34 -35 36 37 0
32 33 34 -35 -36 -37 0
32 33 34 35 -36 37 0
32 33 34 35 36 -37 0
-37 7 18 0
-37 -7 -18 0
37 -7 18 0
37 7 -18 0
-2 4 8 9 0
-2 -4 -8 9 0
-2 -4 8 -9 0
-2 4 -8 -9 0
2 -4 8 9 0
2 -4 -8 -9 0
2 4 -8 9 0
2 4 8 -9 0
3 -38 0
5 -38 0
7 -38 0
38 -3 -5 -7 0
5 -39 0
```

```
7 -39 0
8 -39 0
39 -5 -7 -8 0
3 -40 0
5 -40 0
40 -3 -5 0
5 -41 0
7 -41 0
41 -5 -7 0
7 -42 0
8 -42 0
42 -7 -8 0
-38 39 40 41 42 43 0
-38 -39 -40 41 42 43 0
-38 -39 -40 -41 -42 43 0
-38 -39 -40 -41 42 -43 0
-38 -39 -40 41 -42 -43 0
-38 -39 40 -41 42 43 0
-38 -39 40 -41 -42 -43 0
-38 -39 40 41 -42 43 0
-38 -39 40 41 42 -43 0
-38 39 -40 -41 42 43 0
-38 39 -40 -41 -42 -43 0
-38 39 -40 41 -42 43 0
-38 39 -40 41 42 -43 0
-38 39 40 -41 -42 43 0
-38 39 40 -41 42 -43 0
-38 39 40 41 -42 -43 0
38 -39 40 41 42 43 0
38 -39 -40 -41 42 43 0
38 -39 -40 -41 -42 -43 0
38 -39 -40 41 -42 43 0
38 -39 -40 41 42 -43 0
38 -39 40 -41 -42 43 0
38 -39 40 -41 42 -43 0
38 -39 40 41 -42 -43 0
38 39 -40 41 42 43 0
38 39 -40 -41 -42 43 0
38 39 -40 -41 42 -43 0
38 39 -40 41 -42 -43 0
38 39 40 -41 42 43 0
38 39 40 -41 -42 -43 0
38 39 40 41 -42 43 0
38 39 40 41 42 -43 0
-43 8 19 0
```

```
-43 -8 -19 0
43 -8 19 0
43 8 -19 0
-3 5 9 10 0
-3 -5 -9 10 0
-3 -5 9 -10 0
-3 5 -9 -10 0
3 -5 9 10 0
3 -5 -9 -10 0
3 5 -9 10 0
3 5 9 -10 0
4 -44 0
6 -44 0
8 -44 0
44 -4 -6 -8 0
6 -45 0
8 -45 0
9 -45 0
45 -6 -8 -9 0
4 -46 0
6 -46 0
46 -4 -6 0
6 -47 0
8 -47 0
47 -6 -8 0
8 -48 0
9 -48 0
48 -8 -9 0
-44 45 46 47 48 49 0
-44 -45 -46 47 48 49 0
-44 -45 -46 -47 -48 49 0
-44 -45 -46 -47 48 -49 0
-44 -45 -46 47 -48 -49 0
-44 -45 46 -47 48 49 0
-44 -45 46 -47 -48 -49 0
-44 -45 46 47 -48 49 0
-44 -45 46 47 48 -49 0
-44 45 -46 -47 48 49 0
-44 45 -46 -47 -48 -49 0
-44 45 -46 47 -48 49 0
-44 45 -46 47 48 -49 0
-44 45 46 -47 -48 49 0
-44 45 46 -47 48 -49 0
-44 45 46 47 -48 -49 0
44 -45 46 47 48 49 0
```

```
44 -45 -46 -47 48 49 0
44 -45 -46 -47 -48 -49 0
44 -45 -46 47 -48 49 0
44 -45 -46 47 48 -49 0
44 -45 46 -47 -48 49 0
44 -45 46 -47 48 -49 0
44 -45 46 47 -48 -49 0
44 45 -46 47 48 49 0
44 45 -46 -47 -48 49 0
44 45 -46 -47 48 -49 0
44 45 -46 47 -48 -49 0
44 45 46 -47 48 49 0
44 45 46 -47 -48 -49 0
44 45 46 47 -48 49 0
44 45 46 47 48 -49 0
-49 9 20 0
-49 -9 -20 0
49 -9 20 0
49 9 -20 0
-4 6 10 11 0
-4 -6 -10 11 0
-4 -6 10 -11 0
-4 6 -10 -11 0
4 -6 10 11 0
4 -6 -10 -11 0
4 6 -10 11 0
4 6 10 -11 0
5 -50 0
7 -50 0
9 -50 0
50 -5 -7 -9 0
7 -51 0
9 -51 0
10 -51 0
51 -7 -9 -10 0
5 -52 0
7 -52 0
52 -5 -7 0
7 -53 0
9 -53 0
53 -7 -9 0
9 -54 0
10 -54 0
54 -9 -10 0
-50 51 52 53 54 55 0
```

```
-50 -51 -52 53 54 55 0
-50 -51 -52 -53 -54 55 0
-50 -51 -52 -53 54 -55 0
-50 -51 -52 53 -54 -55 0
-50 -51 52 -53 54 55 0
-50 -51 52 -53 -54 -55 0
-50 -51 52 53 -54 55 0
-50 -51 52 53 54 -55 0
-50 51 -52 -53 54 55 0
-50 51 -52 -53 -54 -55 0
-50 51 -52 53 -54 55 0
-50 51 -52 53 54 -55 0
-50 51 52 -53 -54 55 0
-50 51 52 -53 54 -55 0
-50 51 52 53 -54 -55 0
50 -51 52 53 54 55 0
50 -51 -52 -53 54 55 0
50 -51 -52 -53 -54 -55 0
50 -51 -52 53 -54 55 0
50 -51 -52 53 54 -55 0
50 -51 52 -53 -54 55 0
50 -51 52 -53 54 -55 0
50 -51 52 53 -54 -55 0
50 51 -52 53 54 55 0
50 51 -52 -53 -54 55 0
50 51 -52 -53 54 -55 0
50 51 -52 53 -54 -55 0
50 51 52 -53 54 55 0
50 51 52 -53 -54 -55 0
50 51 52 53 -54 55 0
50 51 52 53 54 -55 0
-55 10 21 0
-55 -10 -21 0
55 -10 21 0
55 10 -21 0
-5 7 11 12 0
-5 -7 -11 12 0
-5 -7 11 -12 0
-5 7 -11 -12 0
5 -7 11 12 0
5 -7 -11 -12 0
5 7 -11 12 0
5 7 11 -12 0
6 -56 0
8 -56 0
```

```
10 -56 0
56 -6 -8 -10 0
8 -57 0
10 -57 0
11 -57 0
57 -8 -10 -11 0
6 -58 0
8 -58 0
58 -6 -8 0
8 -59 0
10 -59 0
59 -8 -10 0
10 -60 0
11 -60 0
60 -10 -11 0
-56 57 58 59 60 61 0
-56 -57 -58 59 60 61 0
-56 -57 -58 -59 -60 61 0
-56 -57 -58 -59 60 -61 0
-56 -57 -58 59 -60 -61 0
-56 -57 58 -59 60 61 0
-56 -57 58 -59 -60 -61 0
-56 -57 58 59 -60 61 0
-56 -57 58 59 60 -61 0
-56 57 -58 -59 60 61 0
-56 57 -58 -59 -60 -61 0
-56 57 -58 59 -60 61 0
-56 57 -58 59 60 -61 0
-56 57 58 -59 -60 61 0
-56 57 58 -59 60 -61 0
-56 57 58 59 -60 -61 0
56 -57 58 59 60 61 0
56 -57 -58 -59 60 61 0
56 -57 -58 -59 -60 -61 0
56 -57 -58 59 -60 61 0
56 -57 -58 59 60 -61 0
56 -57 58 -59 -60 61 0
56 -57 58 -59 60 -61 0
56 -57 58 59 -60 -61 0
56 57 -58 59 60 61 0
56 57 -58 -59 -60 61 0
56 57 -58 -59 60 -61 0
56 57 -58 59 -60 -61 0
56 57 58 -59 60 61 0
56 57 58 -59 -60 -61 0
```

```
56 57 58 59 -60 61 0
56 57 58 59 60 -61 0
-61 11 22 0
-61 -11 -22 0
61 -11 22 0
61 11 -22 0
-6 8 12 13 0
-6 -8 -12 13 0
-6 -8 12 -13 0
-6 8 -12 -13 0
6 -8 12 13 0
6 -8 -12 -13 0
6 8 -12 13 0
6 8 12 -13 0
7 -62 0
9 -62 0
11 -62 0
62 -7 -9 -11 0
9 -63 0
11 -63 0
12 -63 0
63 -9 -11 -12 0
7 -64 0
9 -64 0
64 -7 -9 0
9 -65 0
11 -65 0
65 -9 -11 0
11 -66 0
12 -66 0
66 -11 -12 0
-62 63 64 65 66 67 0
-62 -63 -64 65 66 67 0
-62 -63 -64 -65 -66 67 0
-62 -63 -64 -65 66 -67 0
-62 -63 -64 65 -66 -67 0
-62 -63 64 -65 66 67 0
-62 -63 64 -65 -66 -67 0
-62 -63 64 65 -66 67 0
-62 -63 64 65 66 -67 0
-62 63 -64 -65 66 67 0
-62 63 -64 -65 -66 -67 0
-62 63 -64 65 -66 67 0
-62 63 -64 65 66 -67 0
-62 63 64 -65 -66 67 0
```

```
-62 63 64 -65 66 -67 0
-62 63 64 65 -66 -67 0
62 -63 64 65 66 67 0
62 -63 -64 -65 66 67 0
62 -63 -64 -65 -66 -67 0
62 -63 -64 65 -66 67 0
62 -63 -64 65 66 -67 0
62 -63 64 -65 -66 67 0
62 -63 64 -65 66 -67 0
62 -63 64 65 -66 -67 0
62 63 -64 65 66 67 0
62 63 -64 -65 -66 67 0
62 63 -64 -65 66 -67 0
62 63 -64 65 -66 -67 0
62 63 64 -65 66 67 0
62 63 64 -65 -66 -67 0
62 63 64 65 -66 67 0
62 63 64 65 66 -67 0
-67 12 23 0
-67 -12 -23 0
67 -12 23 0
67 12 -23 0
-7 9 13 14 0
-7 -9 -13 14 0
-7 -9 13 -14 0
-7 9 -13 -14 0
7 -9 13 14 0
7 -9 -13 -14 0
7 9 -13 14 0
7 9 13 -14 0
8 -68 0
10 -68 0
12 -68 0
68 -8 -10 -12 0
10 -69 0
12 -69 0
13 -69 0
69 -10 -12 -13 0
8 -70 0
10 -70 0
70 -8 -10 0
10 -71 0
12 -71 0
71 -10 -12 0
12 -72 0
```

```
13 -72 0
72 -12 -13 0
-68 69 70 71 72 73 0
-68 -69 -70 71 72 73 0
-68 -69 -70 -71 -72 73 0
-68 -69 -70 -71 72 -73 0
-68 -69 -70 71 -72 -73 0
-68 -69 70 -71 72 73 0
-68 -69 70 -71 -72 -73 0
-68 -69 70 71 -72 73 0
-68 -69 70 71 72 -73 0
-68 69 -70 -71 72 73 0
-68 69 -70 -71 -72 -73 0
-68 69 -70 71 -72 73 0
-68 69 -70 71 72 -73 0
-68 69 70 -71 -72 73 0
-68 69 70 -71 72 -73 0
-68 69 70 71 -72 -73 0
68 -69 70 71 72 73 0
68 -69 -70 -71 72 73 0
68 -69 -70 -71 -72 -73 0
68 -69 -70 71 -72 73 0
68 -69 -70 71 72 -73 0
68 -69 70 -71 -72 73 0
68 -69 70 -71 72 -73 0
68 -69 70 71 -72 -73 0
68 69 -70 71 72 73 0
68 69 -70 -71 -72 73 0
68 69 -70 -71 72 -73 0
68 69 -70 71 -72 -73 0
68 69 70 -71 72 73 0
68 69 70 -71 -72 -73 0
68 69 70 71 -72 73 0
68 69 70 71 72 -73 0
-73 13 24 0
-73 -13 -24 0
73 -13 24 0
73 13 -24 0
-8 10 14 15 0
-8 -10 -14 15 0
-8 -10 14 -15 0
-8 10 -14 -15 0
8 -10 14 15 0
8 -10 -14 -15 0
8 10 -14 15 0
```

```
8 10 14 -15 0
-17 0
-18 25 0
18 -25 0
-19 25 0
19 -25 0
-20 0
-21 0
-22 25 0
22 -25 0
-23 25 0
23 -25 0
-24 25 0
24 -25 0
```

B Crypto1 and Hitag2 Diagrams

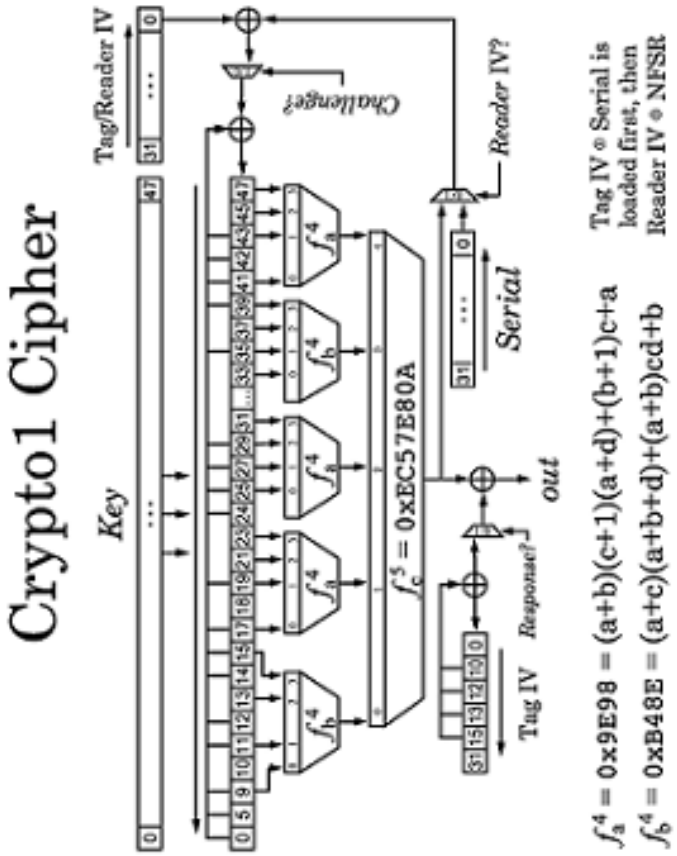


Fig. 3. The stream cipher Crypto1

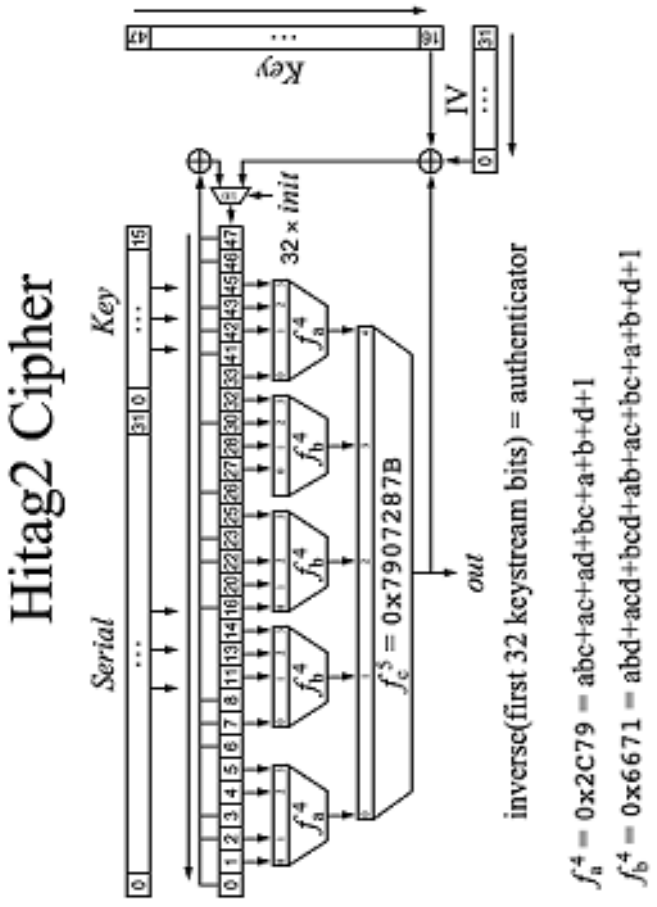


Fig. 4. The stream cipher Hitag2

C Tap Configurations

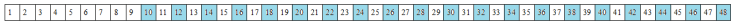


Fig. 5. Crypto1 Taps

Fig. 6. Hitag2 Taps

Fig. 7. "Left" Taps

Fig. 8. "Mid" Taps

Fig. 9. Left Consecutive Taps

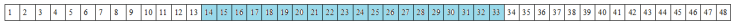


Fig. 10. Mid Consecutive Taps

Fig. 11. Right Consecutive Taps

Fig. 12. "2-2" Taps

Fig. 13. "1-2-1" Taps

Fig. 14. "19-1" Taps

Fig. 15. "10-10" Taps

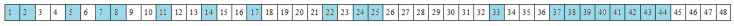


Fig. 16. Random A Taps

Fig. 17. Random B Taps

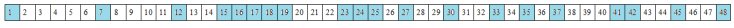


Fig. 18. Random C Taps

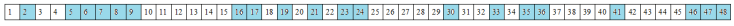


Fig. 19. Random D Taps

Fig. 20. Random E Taps

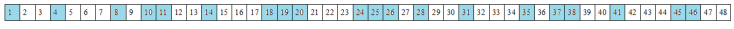


Fig. 21. Random F Taps

D Java Program Source Code

```
import java.util.Random;
import java.util.Scanner;
import java.util.ArrayList;


public class TapConfig{

    // Instance variables
    private ArrayList<Integer> taps;
    private int distScore, diffScore, repScore, distScore1, distScore2;
    private String configName;

    // Static variables
    private static final Integer[] DIST_BASIS1 = {29, 30, 31, 32, 33, 34,
                35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48};
    private static final Integer[] DIST_BASIS2 = {1, 2, 3, 4, 5, 6, 7, 8,
                9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20};
    private static final Integer[] DIST_BASIS3 = {14, 15, 16, 17, 18, 19,
                20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33};

    private static int numTapConfigs = 0;
    private static double sumOfTotalScores = 0;


    // Tap Configurations
    public static final Integer[] RIGHT_CON = {29, 30, 31, 32, 33, 34, 35,
                36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48};
    public static final Integer[] CRYPTO1 = {10, 12, 14, 16, 18, 20, 22,
                24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48};
    public static final Integer[] HITAG2 = {2, 3, 5, 6, 8, 12, 14, 15, 17,
                21, 23, 26, 28, 29, 31, 33, 34, 43, 44, 46};
    public static final Integer[] TWO_TWO = {6, 7, 10, 11, 14, 15, 18, 19,
                22, 23, 26, 27, 30, 31, 34, 35, 38, 39, 42, 43};
    public static final Integer[] ONE_TWO_ONE = {1, 3, 6, 8, 11, 13, 16,
                18, 21, 23, 26, 28, 31, 33, 36, 38, 41, 43, 46, 48};
    public static final Integer[] RAND_A = {1, 2, 5, 7, 8, 11, 14, 17, 22,
                24, 25, 33, 37, 38, 39, 40, 41, 42, 43, 44};
    public static final Integer[] RAND_B = {2, 4, 7, 14, 18, 19, 20, 23, 25,
                26, 27, 28, 29, 34, 36, 39, 40, 41, 42, 46};
    public static final Integer[] RAND_C = {1, 7, 12, 15, 16, 17, 18, 19,
                23, 24, 25, 27, 30, 33, 35, 37, 41, 42, 45, 48};
    public static final Integer[] RAND_D = {2, 5, 6, 7, 8, 9, 16, 17, 19,
                21, 23, 24, 30, 33, 35, 36, 41, 46, 47, 48};
    public static final Integer[] RAND_E = {1, 3, 5, 6, 7, 11, 15, 17, 18,
                23, 26, 36, 37, 38, 40, 41, 43, 44, 46, 47};
    public static final Integer[] RAND_F = {1, 4, 8, 10, 11, 14, 18, 19, 20,
                24, 25, 26, 28, 31, 35, 37, 38, 41, 45, 46};
    public static final Integer[] NINETEEN_ONE = {1, 2, 3, 4, 5, 6, 7, 8, 9,
```

```
            10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 48};
public static final Integer[] TEN_TEN = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
            12, 13, 14, 15, 16, 17, 18, 19, 20, 21};



// Constructor
public TapConfig(String name, Integer[] tapas){
    this.taps = new ArrayList<Integer>(20);
    for(int i = 0;i < 20; i++){
        this.taps.add(i,tapas[i]);
    }
    this.configName = name;
    numTapConfigs++;
    sumOfTotalScores = sumOfTotalScores + totalScore();
}

// Static method

public static Double getAverageScore(){
    return TapConfig.sumOfTotalScores / TapConfig.numTapConfigs;
}

// Random Tap Generator

public static Integer[] randTaps(){
    ArrayList<Integer> ret = new ArrayList(20);
    Integer[] ret2 = new Integer[20];
    Random rand = new Random();
    Integer entry;

    int i = 0;
    while(i < 20){
        entry = rand.nextInt(47);
        if(! ret.contains(entry+1)){
            ret.add(i, entry+1);
            i++;
        }
    }

    for(int j = 0; j < 20; j++){
        ret2[j] = ret.get(j);
    }
    Sort2.insertionSort(ret2);
    return ret2;

}

/** Tap Configuration Subscores **/
```

```
public int getDistanceScore(){
    distScore1 = 0;
    distScore2 = 0;

    for(int i = 0; i < 20; i++){
        distScore1 = distScore1 + (DIST_BASIS1[i] - taps.get(i));
    }
    for(int i = 0; i < 20; i++){
        distScore2 = distScore2 + (taps.get(i) - DIST_BASIS2[i]);
    }

    int minimum = Math.min(distScore1, distScore2);

    return minimum;

}

public Double getDistStanDev(){
    Double distAvg = getDistanceScore()/20.0;
    Double distVar = 0.0;
    int x;
    if(distScore1 < distScore2){
        for(int i = 0; i < 20; i++){
            x = (DIST_BASIS1[i]-taps.get(i));
            distVar = distVar + .05*Math.pow((distAvg-x), 2.0);
        }
    }
    else{
        for(int j = 0; j < 20; j++){
            x = (taps.get(j) - DIST_BASIS2[j]);
            distVar = distVar + .05*Math.pow((distAvg-x), 2.0);
        }
    }
    return Math.sqrt(distVar);
}

public int getDiffusionScore(){
    diffScore = 0;
    for(int i = 1; i < 5; i++){
        for(int j=19; j >=0; j--){
            if(taps.contains(taps.get(j)-i)){
                diffScore++;
            }

        }
    }
    return diffScore;
}

public int getRepetitionScore(){
```

```
ArrayList<Integer> f1, f2, f3, f4, f5;

f1 = new ArrayList<Integer>(4);
f2 = new ArrayList<Integer>(4);
f3 = new ArrayList<Integer>(4);
f4 = new ArrayList<Integer>(4);
f5 = new ArrayList<Integer>(4);

for(int i = 0; i < 4; i++){
    f1.add(taps.get(i));
    f2.add(taps.get(i+4));
    f3.add(taps.get(i+8));
    f4.add(taps.get(i+12));
    f5.add(taps.get(i+16));
}

int f5Score = 0;

for(int j = 0; j < 48; j++){
    int f1Num = 0;
    int f2Num = 0;
    int f3Num = 0;
    int f4Num = 0;
    int f5Num = 0;

    for(int k = 0; k < 4; k++){
        if(f5.contains(f5.get(k)-j)){
            f5Num++;
        }
        else if(f4.contains(f5.get(k)-j)){
            f4Num++;
        }
        else if(f3.contains(f5.get(k)-j)){
            f3Num++;
        }
        else if(f2.contains(f5.get(k)-j)){
            f2Num++;
        }
        else if(f1.contains(f5.get(k)-j)){
            f1Num++;
        }
    }

    if(f1Num == 1){
        f1Num = 0;
    }
    if(f2Num == 1){
        f2Num = 0;
    }
    if(f3Num == 1){
```

```
        f3Num = 0;
    }
    if(f4Num == 1){
        f4Num = 0;
    }
    if(f5Num ==1){
        f4Num = 0;
    }

    f5Score = f5Score + f1Num + f2Num + f3Num + f4Num + f5Num;
}

int f4Score = 0;

for(int j = 0; j < 44; j++){
    int f1Num = 0;
    int f2Num = 0;
    int f3Num = 0;
    int f4Num = 0;

    for(int k = 0; k < 4; k++){
       if(f4.contains(f4.get(k)-j)){
            f4Num++;
       }
       else if(f3.contains(f4.get(k)-j)){
            f3Num++;
       }
       else if(f2.contains(f4.get(k)-j)){
            f2Num++;
       }
       else if(f1.contains(f4.get(k)-j)){
            f1Num++;
       }
    }

    if(f1Num == 1){
        f1Num = 0;
    }
    if(f2Num == 1){
        f2Num = 0;
    }
    if(f3Num == 1){
        f3Num = 0;
    }
    if(f4Num == 1){
        f4Num = 0;
    }

    f4Score = f4Score + f1Num + f2Num + f3Num + f4Num;
}
```

```
int f3Score = 0;

for(int j = 0; j < 40; j++){{ }
    int f1Num = 0;
    int f2Num = 0;
    int f3Num = 0;

    for(int k = 0; k < 4; k++){
       if(f3.contains(f3.get(k)-j)){
            f3Num++;
       }
       else if(f2.contains(f3.get(k)-j)){
            f2Num++;
       }
       else if(f1.contains(f3.get(k)-j)){
            f1Num++;
       }
    }

    if(f1Num == 1){
        f1Num = 0;
    }
    if(f2Num == 1){
        f2Num = 0;
    }
    if(f3Num == 1){
        f3Num = 0;
    }


    f3Score = f3Score + f1Num + f2Num + f3Num;
}

int f2Score = 0;

for(int j = 0; j < 36; j++){
    int f1Num = 0;
    int f2Num = 0;

    for(int k = 0; k < 4; k++){
      if(f2.contains(f2.get(k)-j)){
            f2Num++;
      }
      else if(f1.contains(f2.get(k)-j)){
            f1Num++;
      }
    }

    if(f1Num == 1){
```

```
            f1Num = 0;
        }
        if(f2Num == 1){
            f2Num = 0;
        }

        f2Score = f2Score + f1Num + f2Num;
    }

    int f1Score = 0;

    for(int j = 0; j < 36; j++){
        int f1Num = 0;

        for(int k = 0; k < 4; k++){
          if(f1.contains(f1.get(k)-j)){
                f1Num++;
          }
        }

        if(f1Num == 1){
            f1Num = 0;
        }

        f1Score = f1Score + f1Num;
    }
    return f5Score+f4Score+f3Score+f2Score+f1Score;
}

public Double spaceCount(){
    Integer[] xArray =
            {0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
                0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0};
    Double xNum = 0.0;
    for(int i = 0; i< 19; i++){
        xArray[taps.get(i+1)-taps.get(i)-1] =
                xArray[taps.get(i+1)-taps.get(i)-1] + 1;
    }
    for(Integer j: xArray){
        if(j != 0){
            xNum = xNum + 1.0;
        }
    }
   return xNum;
}

public Double spaceDev(){
    int sum = 0;
    Double num = 0.0;
    for(int i = 0; i< 19; i++){
```

```
            int x = (taps.get(i+1)-taps.get(i));
            if(x != 0){
                sum = sum + x;
                num = num + 1.0;
            }
        }
        Double avg = sum/num;
        Double variance = 0.0;
        for(int i = 0; i< 19; i++){
            int x = (taps.get(i+1)-taps.get(i));
            if(x != 0){
                variance = variance+(1.0/num)*Math.pow((x-avg), 2.0);
            }
        }
        return Math.sqrt(variance);

    }

    public Double spanDev(){
        boolean connected = true;
        Integer[] spans= {0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0};
        int i = 0;
        int j = 0;
        int run = 1;
        while((i < 19)){
            if((taps.get(i+1) - taps.get(i)) == 1){
                run++;
                i++;
            }
             else{
                 spans[j] = run;
                 run = 1;
                 i++;
                 j++;
             }

        }
        spans[j] = run;
        int sum = 0;
        Double num = 0.0;
        for(Integer span : spans){
            if(span != 0){
                sum = sum + span;
                num = num + 1.0;
             }
        }
        Double avg = sum/num;
        Double variance = 0.0;
        for(Integer k: spans){
             if(k != 0){
```

```
            variance = variance+(1.0/num)*Math.pow((k - avg),2.0);
        }
    }

    return Math.sqrt(variance);

}

 public Double totalScore(){
     return 8.05 + (.014 * this.getDistanceScore())
             - (.0939 * this.getDiffusionScore())
             - (.0081 * this.getRepetitionScore())
             + (.6 * spaceCount() + 1.1 * spaceDev()
             -(.573 * spanDev()));

 }

 /** Display Methods**/

 public String toString(){
     String str;

     str = "====================== \nName: " + configName
             + "\nDistance Score: " + this.getDistanceScore()
             + "\nDiffusion Score: " + this.getDiffusionScore()
             + "\nRepetition Score: " + this.getRepetitionScore()
             + "\nSpacing Count Standard Dev: " + spaceCount()
             + "\nSpace Dev: " + spaceDev() + "\nSpan Standard Dev: "
             + spanDev() + "\nDist Stand Dev: " + getDistStanDev()
             + "\nTotal Score: " + this.totalScore()
             + "\n======================\n";

     return str;
 }

 public String toString2(){
     String str;

     str = "======================\n[ ";

     for(int i = 0; i < 19; i++){
         str = str + this.taps.get(i) + ", ";
     }

     str = str + this.taps.get(19) + "]";

     str = str + "\nTotal Score: " + this.totalScore()
             + "\n======================\n";

     return str;
```

```
}

/** The main test methods**/

// Random Tests

public static void runRandSample(){
    Scanner in;
    TapConfig rand;
    int n;
    int x;
    double maxVal = 0;
    TapConfig maxTaps = new TapConfig("Maximum", TapConfig.randTaps());
    ArrayList<TapConfig> maxes = new ArrayList<TapConfig>();

    in = new Scanner(System.in);
    System.out.print("Enter the number of random
                tap configurations you want to test: ");
    n = in.nextInt();
    System.out.println();
    System.out.print("Enter 1 for keeping track, 2 for no: ");
    x = in.nextInt();


    for(int i = 0; i < n; i++){
        String name = "Random " + i;
        rand = new TapConfig(name, TapConfig.randTaps());

        if(rand.totalScore() > maxVal){
            maxTaps = rand;
            maxVal = rand.totalScore();
        }
        if(((i % 100) == 0) && (x == 1)){
            maxes.add(maxTaps);
        }
        if((x == 2) && ((i % 1000) == 0)){
            System.out.println((i/((double)n))*100 + " %");
        }
    }

    for(TapConfig m : maxes){
        System.out.println(m.totalScore());
    }

    System.out.println( maxTaps.toString2());
}

// Our basic configs
```

```
    public static void standardScores(){
        TapConfig rightCon = new TapConfig("Right Con", TapConfig.RIGHT_CON);
        TapConfig crypto1 = new TapConfig("Crypto1", TapConfig.CRYPTO1);
        TapConfig hitag2 = new TapConfig("Hitag2", TapConfig.HITAG2);
        TapConfig oneTwoOne = new TapConfig("1-2-1 Taps", TapConfig.ONE_TWO_ONE);
        TapConfig twoTwo = new TapConfig("2-2 Taps", TapConfig.TWO_TWO);
        TapConfig randA = new TapConfig("Random A", TapConfig.RAND_A);
        TapConfig randB = new TapConfig("Random B", TapConfig.RAND_B);
        TapConfig randC = new TapConfig("Random C", TapConfig.RAND_C);
        TapConfig randD = new TapConfig("Random D", TapConfig.RAND_D);
        TapConfig randE = new TapConfig("Random E", TapConfig.RAND_E);
        TapConfig randF = new TapConfig("Random F", TapConfig.RAND_F);
        TapConfig nineteenOne = new TapConfig("19-1 Taps", TapConfig.NINETEEN_ONE);
        TapConfig tenTen = new TapConfig("10-10 Taps", TapConfig.TEN_TEN);

        System.out.println(rightCon);
        System.out.println(crypto1);
        System.out.println(hitag2);
        System.out.println(oneTwoOne);
        System.out.println(twoTwo);
        System.out.println(randA);
        System.out.println(randB);
        System.out.println(randC);
        System.out.println(randD);
        System.out.println(randE);
        System.out.println(randF);
        System.out.println(nineteenOne);
        System.out.println(tenTen);
    }


}
```