SPSU Math 1113: Precalculus Cheat Sheet

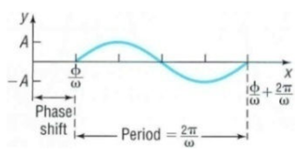
§5.1 Polynomial Functions and Models (review)
Steps to Analyze Graph of Polynomial
1. y-intercepts: $f(0)$
2. x-intercept: $f(x) = 0$
3. f crosses / touches axis @ x-intercepts
4. End behavior: like leading term
5. Find max num turning pts of f: $(n - 1)$
6. Behavior near zeros for each x-intercept
7. May need few extra pts to draw fcn.

§5.2 Rational Functions
Finding Horizontal/Oblique Asymptotes of R
where degree of numer. = n and degree of denom. = m
1. If n < m, horizontal asymptote: $y = 0$ (the x-axis).
2. If n = m, line $y = \frac{a_n}{b_m}$ is a horizontal asymptote.
3. If n = (m + 1), quotient from long div is $ax + b$ and line $y = ax + b$ is oblique asymptote.
4. If n > (m + 1), R has no asymptote.

§7.6 Graphing Sinusoidals
Graphing y = A sin (ωx) & y = A cos (ωx)
|A| = amplitude (stretch/shrink vertically)
$|A| < 1$ shrink $|A| > 1$ stretch $A < 0$ reflect
Distance from min to max = 2A

ω = frequency (stretch/shrink horizontally)
$|\omega| < 1$ stretch $|\omega| > 1$ shrink $\omega < 0$ reflect
period = $T = \left|\frac{2\pi}{\omega}\right|$

§7.8 Phase Shift = $\frac{\varphi}{\omega}$
$y = A\sin(\omega x - \varphi) + B$
$y = A\cos(\omega x - \varphi) + B$



§8.1 Inverse Sin, Cos, Tan Fcns
$y = \sin^{-1}(x)$ Restrict range to $[-\pi/2, \pi/2]$
$y = \cos^{-1}(x)$ Restrict range to $[0, \pi]$
$y = \tan^{-1}(x)$ Restrict range to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

§8.2 Inverse Trig Fcns (con't)
$y = \sec^{-1} x$ where $|x| \geq 1$ and $0 \leq y \leq \pi$, $y \neq \frac{\pi}{2}$
$y = \csc^{-1} x$ where $|x| \geq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, $y \neq 0$
$y = \cot^{-1} x$ where $-\infty < x < \infty$ and $0 < y < \pi$

§8.3 Trig Identities
$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$
$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$
Pythagorean: $\sin^2\theta + \cos^2\theta = 1$
$\tan^2\theta + 1 = \sec^2\theta$ $\cot^2\theta + 1 = \csc^2\theta$

§8.4 Sum & Difference Formulae
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

§8.5 Double-Angle & Half-Angle Formulae
$\sin(2\alpha) = 2\sin\alpha\cos\alpha$ $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$
$\cos(2\alpha) = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$

$\tan(2\alpha) = \frac{2\tan(\alpha)}{1 - \tan^2(\alpha)}$

$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}}$ $\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$

$\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{1+\cos(\alpha)}} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$

§9.2 Law of Sines
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

§9.3 Law of Cosines
$a^2 = b^2 + c^2 - 2bc\cos A$
$c^2 = a^2 + b^2 - 2ab\cos C$ $b^2 = a^2 + c^2 - 2ac\cos B$

§9.4 Area of Triangle
$K = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$
Heron's Formula $s = \frac{1}{2}(a + b + c)$
$K = \sqrt{s(s-a)(s-b)(s-c)}$

§9.5 Simple & Damped Harmonic Motion
Simple Harmonic Motion
$d = a\cos(\omega t)$ or $d = a\sin(\omega t)$
Damped Harmonic Motion
$d(t) = ae^{-(bt)/(2m)}\cos\left(\sqrt{\omega^2 - \frac{b^2}{4m^2}}\ t\right)$

where a, b, m constants:
b = *damping factor (damping coefficient)*
m = mass of oscillating object
$|a|$ = displacement at t = 0
$\frac{2\pi}{\omega}$ = period if no damping

§10.1 Polar Coordinates
Convert Polar to Rectangular Coordinates
$x = r\cos\theta$ $y = r\sin\theta$
Convert Rectangular to Polar Coordinates
If $x = y = 0$ then $r = 0$, θ can have any value
else $r = \sqrt{x^2 + y^2}$

$$\theta = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & Q_I \text{ or } Q_{IV} \\ \tan^{-1}\left(\frac{y}{x}\right) + \pi & Q_{II} \text{ or } Q_{III} \\ \pi/2 & x = 0,\ y > 0 \\ -\pi/2 & x = 0,\ y < 0 \end{cases}$$

§10.3 Complex Plane & De Moivre's Theorem
Conjugate of $z = x + yi$ is $\bar{z} = x + yi$
Modulus of z: $|z| = \sqrt{z\,\bar{z}} = \sqrt{x^2 + y^2}$
Products & Quotients of Complex Nbs (Polar)
$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$ $z_2 \neq 0$-

De Moire's Theorem $z = r(\cos\theta + i\sin\theta)$
$z^n = r^n[\cos(n\theta) + i\sin(n\theta)]$ $n \geq 1$

Complex Roots $n \geq 2$, $k = 0, 1, 2, \ldots, (n-1)$)
$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right]$
where $k = 0, 1, 2, \ldots, (n-1)$

§10.4 Vectors

Unit Vectors

unit vectors: **i, j, k** in direction x-axis, y-axis, z-axis

Add & Subtract Vectors Algebraically

$\boldsymbol{v} = (a_1, b_1) = a_1\mathbf{i} + b_1\mathbf{j}$ $\quad \boldsymbol{w} = (a_2, b_2) = a_2\mathbf{i} + b_2\mathbf{j}$ ⇨

$\boldsymbol{v} + \boldsymbol{w} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j} = (a_1 + a_2, b_1 + b_2)$

$\boldsymbol{v} - \boldsymbol{w} = (a_1 - a_2)\mathbf{i} + (b_1 - b_2)\mathbf{j} = (a_1 - a_2, b_1 - b_2)$

$\alpha \boldsymbol{v} = (\alpha a_1)\mathbf{i} + (\alpha b_1)\mathbf{j} = (\alpha a_1, \alpha b_1)$

$\|\boldsymbol{v}\| = \sqrt{{a_1}^2 + {b_1}^2}$

§10.5 The Dot Product

$\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ $\quad \mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$

⇨ $\mathbf{v} \cdot \mathbf{w} = a_1 a_2 + b_1 b_2$

Angle between 2 Vectors $\quad \cos\theta = \dfrac{\boldsymbol{u}\cdot\boldsymbol{v}}{\|\boldsymbol{u}\|\|\boldsymbol{v}\|}$

Decompose a Vector into Orthogonal Vectors

Vector projection of $\boldsymbol{v}$ onto $\boldsymbol{w}$ $\quad \boldsymbol{v_1} = \dfrac{\boldsymbol{u}\cdot\boldsymbol{w}}{\|\boldsymbol{w}\|^2}\boldsymbol{w}$

$\boldsymbol{v_2} = \boldsymbol{v} - \boldsymbol{v_1}$

- Draw $\boldsymbol{v}$ & $\boldsymbol{w}$ with same initial pt
- From terminal pt of $\boldsymbol{v}$ drop $\perp$ to $\boldsymbol{w}$
- This creates rt triangle with $\boldsymbol{v}$ as hypotenuse.
- Legs of triangle are decomposition

§12.1 Sys of Linear Eqns; Substitution/Elimination

Solve Systems of Equations by Substitution

1. Solve 1 eqn for 1 variable in terms of others.
2. Substitute result in remaining eqns.
3. If have eqn in 1 variable, solve it, otherwise loop back to 1 above.
4. Solve remaining variables, if any, by substituting known values in remaining eqns.
5. Check soln in original system of eqns.

Solve Systems of Eqns by Elimination

1. Interchange any 2 eqns.
2. Multiply (or divide) each side of eqn by same non-zero constant.
3. Replace any eqn in system by sum (or difference) of that eqn & nonzero multiple of another eqn in system.

§12.2 Systems of Linear Eqns: Matrices

Row Operations on the Matrix:

1. Interchange any 2 rows.
2. Replace a row by nonzero multiple of that row.
3. Replace a row by sum of that row and a nonzero multiple of some other row.

Matrix Method for Solving System Linear Eqns

1. Write augmented matrix that represents the system.
2. Perform row operations that place "1" in locn 1, 1: Perform row operations that place "0" below this.
3. Perform row operations that place "1" in locn 2, 1, leaving entries to left unchanged. If this is not possible, move 1 cell to right and try again. Perform row operations that place "0" below it & to left.
4. Repeat step 4, moving one row down and 1 col right. Repeat until bottom row or vertical bar reached.
5. Now in row echelon form. Analyze resulting system of eqns for solns to original system of eqns.

§12.3 Systems of Linear Eqns: Determinants

$\begin{cases} ax + by = s \\ cx + dy = t \end{cases}$ $\quad D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc) \neq 0$

$D_x = \begin{vmatrix} s & b \\ t & d \end{vmatrix}$ $\quad D_y = \begin{vmatrix} a & s \\ c & t \end{vmatrix}$

Cramer's Rule: $\quad x = \frac{D_x}{D} \quad y = \frac{D_y}{D}$ etc.

$\begin{cases} a_{11}x + a_{12}y + a_{13}z = c_1 \\ a_{21}x + a_{22}y + a_{23}z = c_2 \\ a_{31}x + a_{32}y + a_{33}z = c_3 \end{cases}$

$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$

the unique soln of system given by

$x = \frac{D_x}{D} \quad y = \frac{D_y}{D} \quad z = \frac{D_z}{D}$

Properties of Determinates

Value of D changes sign if 2 rows interchanged.
Value of D changes sign if 2 columns interchanged.
If all entries in any row are zero, then D = 0
If all entries in any column are zero, then D = 0
If any 2 rows have identical corresponding values then D = 0
If any 2 columns have identical corresponding values then D = 0
If any row multiplied by (nonzero) number k, D is multiplied by k.
If any column multiplied by (nonzero) k, D is multiplied by k.
If entries of any row multiplied by nonzero k and result added to corresponding entries of another row, value of D is unchanged.
If entries of any column multiplied by nonzero k and result added to corresponding entries of another column, D is unchanged.

§12.4 Matrix Algebra

Product of Row x Column:

$$RC = [r_1 r_2 \ldots r_n]\begin{bmatrix} c_1 \\ c_2 \\ \ldots \\ c_n \end{bmatrix} = r_1c_1 + r_2c_2 + \cdots + r_nc_n$$

Product of rectangular matrices:

A is m x r matrix, $\quad B$ is r x n matrix.

$A_{ij} = \Sigma_k A_{ik} B_{kj}$

Finding Inverse of Nonsingular Matrix

To find inverse of n x n nonsingular matrix A:

1. Form the matrix $[A \mid I_n]$.
2. Transform $[A \mid I_n]$ into *reduced* row echelon form.
3. Reduced row echelon form of $[A \mid I_n]$ will contain identity matrix I_n left of vertical bar; the n x n matrix on right of vertical bar is inverse of A.

Solve System Linear Eqns Using Inverse Matrix

Can write system of eqns as AX = B.

If have inverse A^{-1} then multiply by it.

$X = A^{-1} B$

§12.6 Matrix Algebra

Solving by Substitution

For system of eqns, pts whose coordinates satisfy all eqns are represented by intersections of the graphs of eqns.
Can also use substitution & or elimination just like systems of linear eqns.
Beware of extraneous solns.