Digital Simulation of Thunder from Three-Dimensional Lightning

James Dunkin - Wittenberg University
Project Advisor – Dr. Daniel Fleisch

Abstract

The physics of lightning and its resultant thunder have been investigated by many people, but we still don't have a full understanding of the governing processes. In this study, we have constructed a three-dimensional model of lightning using MATLAB® software, and used N-waves as postulated by Ribner and Roy to synthesize the resultant thunder signature. In addition, we have taken an FFT of the thunder signature, and compared the time-domain waveform and frequency spectrum to recordings of thunder taken over the summer of 2009. This analysis is done with the goal of further understanding the processes of thunder production.

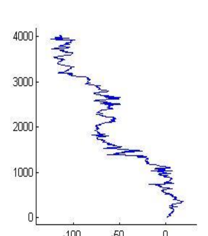
Theory of lightning and thunder

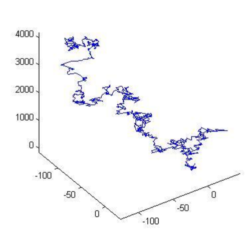
Lightning, at the most basic level, is a discharge of electricity. The charge is built as water droplets and small hailstones collide in the cloud, transferring electrons between themselves. Generally, a net negative charge accumulates at the bottom of a storm cloud, with an accompanying net positive charge at the top. The potential difference between the cloud and the Earth is on the order of 10^8 V. During the discharge of the lightning bolt, current is usually around 30 to 100 kA, but has been recorded as large as 300 kA.

The currently accepted theory for the formation of thunder from lightning is thermal expansion. The lightning heats up the air up to 30,000 K, causing the air around the lighting channel to expand very rapidly. This rapid expansion raises the air pressure in the immediate area and creates a shockwave, which dissipates into an acoustic wave that we hear as thunder. Despite the large amount of energy left in the acoustic wave, only about 1% of the shockwave's original energy is left, with all the rest going into heating of the atmosphere.

Digital lightning

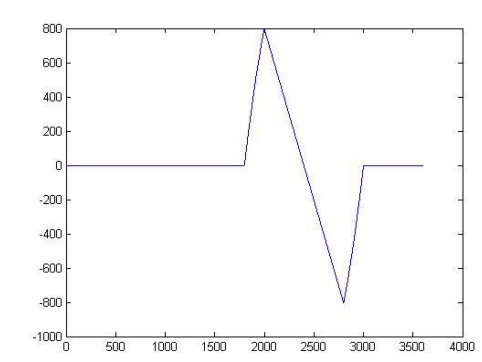
To attempt to recreate lightning, we used MATLAB® software. Starting from the strike location on the ground, the lightning channel was built upwards in segments of average length 3 meters, with a normalized random distribution around the average. Both the zenith and azimuth angles were randomly generated. The zenith angle was restricted to a cone of 30^0, and biased towards the last 4 values of this angle, while the azimuth angle was allowed to be any value.



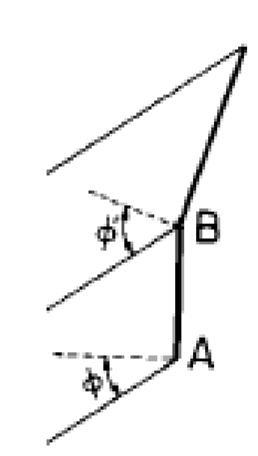


N-waves

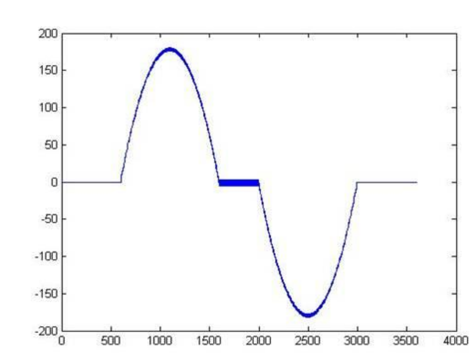
N-waves are the solution to creating thunder out of the digital thunder. A N-wave is the summation of two parabolic pressure waves, one positive and one negative, emitted from a short spark. The overall shape of the N-wave is dependent on the observed angle from the normal of the segment.



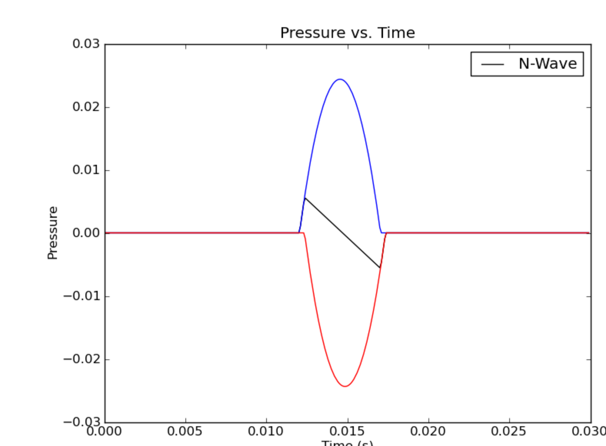
small angle



$$\psi = \frac{cT}{l} \qquad \tau = \frac{(ct) - r}{l} \qquad B = \frac{Al^2}{2rcT}$$



large angle



$$for \sin(\theta) \geq \psi$$

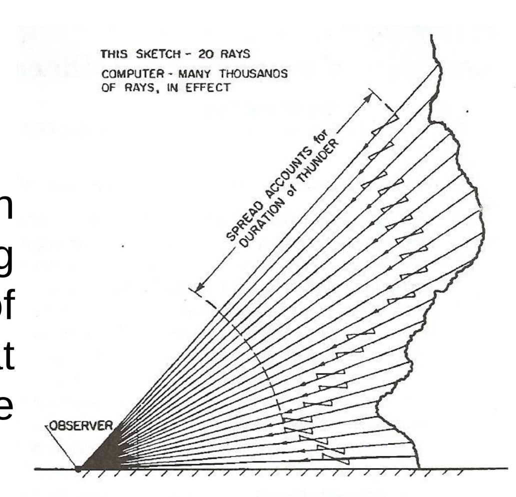
$$\begin{aligned} p &= 0 \; when \; \tau < \frac{2}{3}\psi - \sin(\theta) \\ &= -\frac{3}{4}\frac{B\psi}{\sin(\theta)}(\tau - 2\psi + \sin(\theta) \; when \; \frac{2}{3}\psi \leq \tau < 2\psi - \sin(\theta) \\ &= 0 \; when \; 2\psi - \sin(\theta) \leq \tau < \sin(\theta) \\ &= \frac{B}{\sin(\theta)}(\tau - \sin(\theta))[(\tau - \sin(\theta)) - 2\psi] \; when \sin(\theta) \leq \tau < 2\psi + \sin(\theta) \\ &= 0 \; when \; \tau \geq 2\psi + \sin(\theta) \end{aligned}$$

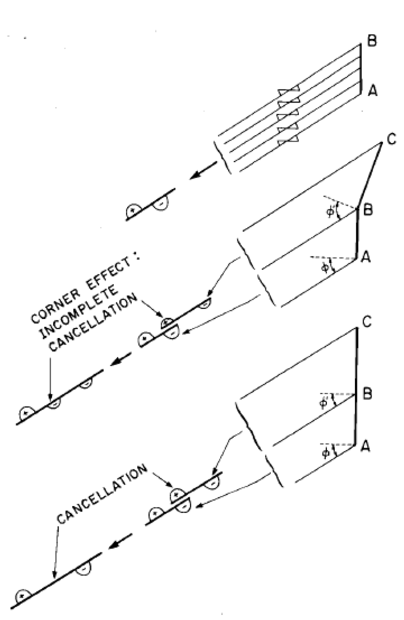
$$for \sin(\theta) < \psi$$

$$\begin{aligned} p &= 0 \; when \; \tau < 0 \\ &= 4B(\tau - \psi) \; when \; 0 \leq \tau < 2\psi - \sin(\theta) \\ &= \frac{B}{\sin(\theta)}(\tau - \sin(\theta))[(\tau - \sin(\theta)) - 2\psi] \; when \; 2\psi - \sin(\theta) \leq \tau < 2\psi + \sin(\theta) \\ &= 0 \; when \; \tau \geq 2\psi + \sin(\theta) \end{aligned}$$

Digital thunder

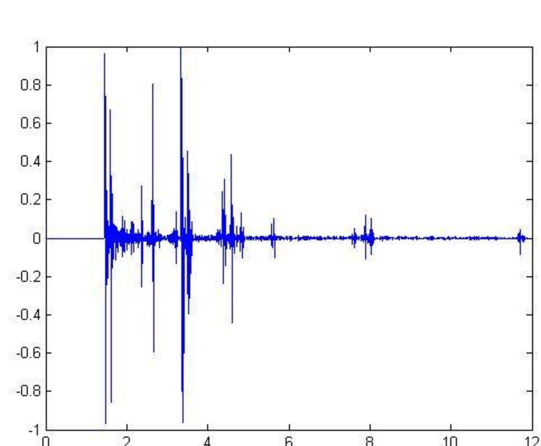
In creating digital thunder, each segment from the digital lightning bolt emits N-waves. Each of those waves is then summed at each point in time to obtain the final thunder signature.





The digital thunder signature is determined by the relation of adjacent segments of the digital lightning. If two segments are parallel, then the N-waves from them completely cancel in the middle, leaving two parabolic pressure waves with a large gap in between them. If two segments have an angle between them, then there will be incomplete cancellation, and there will be a larger pressure wave, and two smaller parabolas of the opposite orientation.

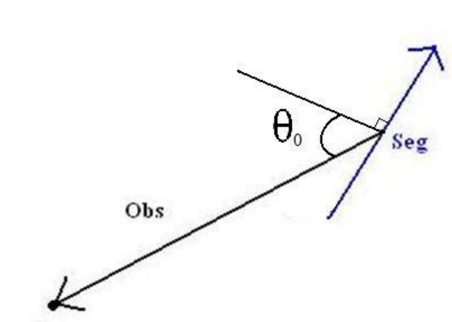
The spikes in the time domain come from regions of the lightning that are parallel with the observer's line to those segments. The lower amplitude 'rumbling' features come from regions of the lightning whose segments are at some angle to the observer's line of sight. The larger the angle, the smaller the amplitude will be.

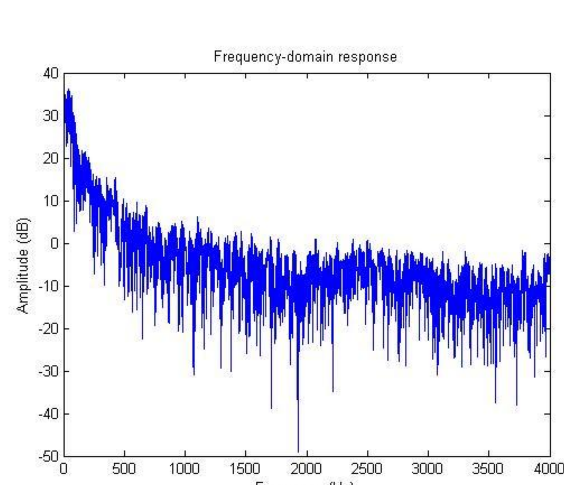


Observer angle

To calculate the N-waves for each segment, the angle between the normal of the segment and a line to the observer is needed. Obs is a vector from the midpoint of the segment (seg) to the observer.

$$\theta_o = cos^{-1}\left(\frac{seg \circ obs}{\sqrt{seg \circ seg} * \sqrt{obs \circ obs}}\right) - \frac{\pi}{2}$$





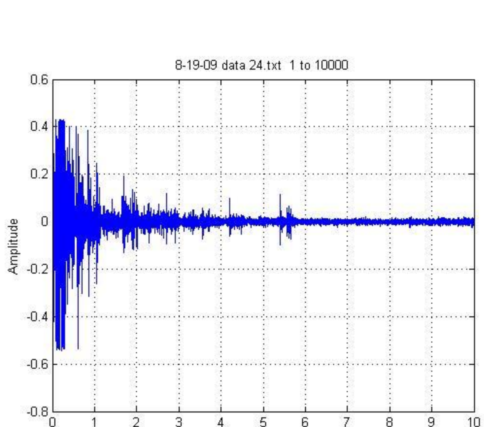
To get the frequency spectrum, I took an FFT of the time domain. The majority of the spectral information is under 500 Hz.

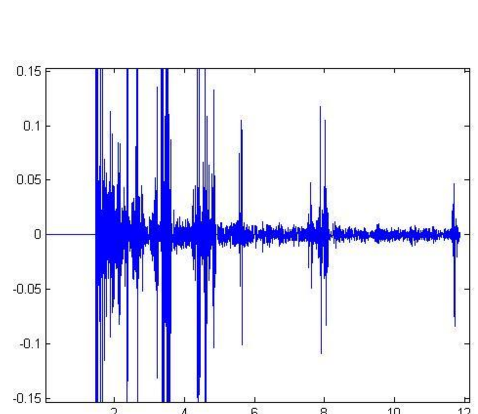
Recording thunder

Over the summer of 2009, I recorded thunder from several thunderstorms. The data was saved as a text file, encoding the time, pressure, and frequency content after a FFT. I used a MATLAB® program to recreate both the time domain and the frequency spectrum graphs from the text data.

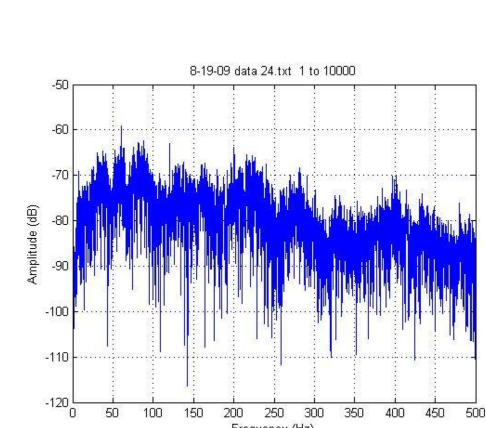
Conclusions

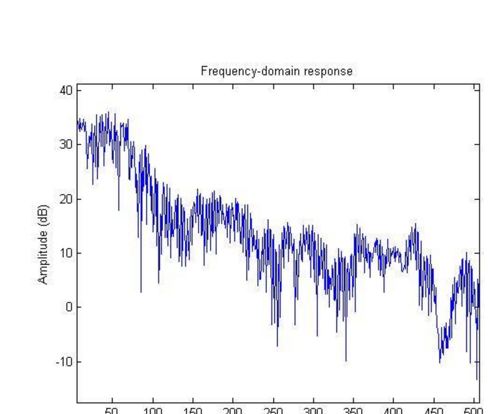
Thunder calculated from three-dimensional lightning gives reasonable results when compared to natural thunder. There are additional modifications to this model that may improve the correlation. These include, but are not limited to: branching of the lightning path, atmospheric diffraction, a more sophisticated attenuation model, and placing limitations on the azimuth angle similar to those on the zenith angle.





Natural thunder Time domain Digital thunder





Natural thunder Frequency spectrum Digital thunder

I compared both the time domains and frequency spectra of my digital thunder and the thunder recorded over the summer of 2009.

The slopes of the upper 400 Hz agree to within 20%, and the widths of minor features of the spectra are 25-40Hz for both natural and digital thunder.

References

1) F. Blanco, P. La Rocca, C. Petta, and F. Riggi. *Modelling digital thunder.* Eur. J. Phys. **30** (2009)

2) D. Roy. *A Monte Carlo Model of Tortuous Lightning and the Generation of Thunder*. UTIAS, Rep. 243 (1981)

3) D. Roy and H.S. Ribner. *Acoustics of Thunder: A quasilinear model for tortuous lightning.* J. Acoust. Soc. Am. **72**(6), (1982)